Module 5.1

Slide 1: This lecture series will focus on equatorial waves, and a special case of equatorial disturbances, the Madden-Julian Oscillation. The first module will take us through the theoretical derivation of several types of equatorial waves and show some examples of how these disturbances can be tracked through the tropics.

Slide 2: As we've stated a few times during the course now, the key difference between mid-latitude and tropical dynamics is that the Coriolis parameter is small in the tropics and goes to zero at the equator. This means that a minimum in absolute vorticity climatologically exists at the equator. This minimum acts as a waveguide along which equatorial disturbances can become trapped.

Slide 3: We will use a single-layer, or barotropic, shallow water model to approximate waves on a beta-plane at the equator. The beta-plane means that we approximate the Coriolis parameter as a linear function of latitude. The set of horizontal momentum and continuity equations for this system is shown on the right. H-bar represents the mean depth of the fluid, while h-prime represents a deviation from this depth.

Slide 4: To find wave solutions, we first linearize relative to a mean background state at rest. Therefore, u-bar and v-bar are 0. We can also re-express the fluid depth with geopotential as shown here. Note in the continuity equation that h-bar has been replaced by the square of the gravity wave phase speed in the shallow water model after multiplying the equation by *g*.

Slide 5: We will assume wave-like solutions in the zonal direction that vary in time. These solutions have some as of yet unspecified structure in the latitudinal direction, which is denoted by the carets above the variables. If we substitute these waveforms into our linearized momentum and continuity equations, we get the following equation for v-hat when eliminating u-hat and phi-hat. For convenience, we can choose to rescale the equations such that we measure length in units of *L* and time in units of *T*. The length scale *L* is related to the Rossby radius and describes the latitudinal spatial scale across which the equatorial gravity wave solutions do not significantly feel the effects of Earth's rotation. After we do the scaling, our solution for v-hat is simplified because we have made β and *c* go to 1.

Slide 6: This equation requires some sort of boundary condition in the latitudinal direction. Since we are concerned with equatorial disturbances only, we can set the condition that *v* goes to zero as y becomes large. With this boundary condition, the equation simply becomes Schrödinger's equation for a harmonic oscillator, and the known solution to this equation requires the relationship between frequency and wavenumber that becomes our general dispersion relation for equatorial waves. The meridional structure of our solutions takes on the form of parabolic cylinder functions that contain Hermite polynomials.

Slide 7: Starting with this dispersion relation, we can then take the limit for a few different scenarios. First, taking the limit for low frequency, such that frequency-squared is much less than frequency, gives us the following dispersion relationship. Note that without scaling, we can clearly see how the wave propagation depends on β. This solution represents a family of equatorial Rossby wave solutions, each with a different positive value of *n*. Recall the general equations for phase speed and group velocity, noting that if they are not the same, the wave is dispersive. Clearly, this is the case for the Rossby wave. The equatorial Rossby wave will propagate westward, while its group velocity is toward the east.

Slide 8: The equatorial Rossby wave is characterized by two off-equatorial cyclonic and anticyclonic gyres that are symmetric about the equator. Recall the near-equatorial response of the circulation to symmetric equatorial heating from a different module, and you can now see how the cyclonic gyres centered to the west of the heating represented the Rossby wave component of the circulation response. The vectors in the figure represent the wave circulation. There is no cross-equatorial flow in the Rossby wave; however, there is diffluent flow at its center. The circulation pattern is characterized by rotation around each of the gyres, with the magnitude of the wind decaying at higher latitude.

Slide 9: If we go back to our dispersion relationship and now take the limit for large frequency, we now get a family of solutions that looks like equatorially trapped Poincaré waves. For large wavenumbers, the second term in the dispersion relationship dominates, and this solution behaves more like a pure gravity wave. This family of solutions is known as the inertio-gravity waves, and they can propagate eastward or westward. These waves are also dispersive.

Slide 10: A couple of examples of westward propagating inertio-gravity waves are shown here for two values of *n*. For odd numbered *n*, the solution is symmetric about the equator, and for even numbered solutions, they are asymmetric reflections. Inertio-gravity waves are characterized by a meridional wavenumber that increases as *n* increases and by cross-equatorial flow in the even-numbered modes. The different shading can be thought of as where anomalies of outgoing longwave radiation might be found in a wave. The striped shading represents positive anomalies, or reduced convection, and the solid anomalies represent negative OLR anomalies, or enhanced convection. This same convention will also be used in similar plots shown shortly. They tend to be limited in spatial scale compared to other wave solutions. Eastward propagating inertio-gravity waves with non-zero *n* share similar structure but have a positive zonal wavenumber *k*.

Slide 11: We can also examine the special case of our dispersion relation such that *n* = 0. When we do this, the general dispersion relation simplifies to an expression that consists of two factors. The simplified relation obviously has three roots. One of these is that ω equals *k*; however, we must discard this solution because we implied that $\omega - k \neq 0$ when deriving our equation for v-hat. This leaves us with a quadratic solution for the frequency, which looks like the expression in the red box had we not scaled. The effects of both rotation and buoyancy are clear in this expression as well. If we take the positive root, our solution is the n=0 eastward propagating inertio-gravity wave. If we take the negative root, we get non-unique solutions that vary with *k* that include both Rossby and westward propagating inertio-gravity wave components. Together, this is considered the mixed Rossby-gravity wave.

Slide 12: The mixed Rossby-gravity wave is characterized by off-equatorial convective maxima and minima. Convective maxima are located to the west of geopotential minima. Cross-equatorial flow is present such that the meridional flow is strongest at the equator where the zonal flow is zero. The wave circulation consists of two gyres that are centered on the equator.

Slide 13: The n=0 eastward propagating inertio-gravity wave has a similar geopotential and convective structure; however, the circulation is subtly different than the mixed Rossby-gravity wave in that the sign of the meridional wind is reversed.

Slide 14: One additional special case exists that is separate from the dispersion relation we have seen. It occurs when the meridional wind in the wave is zero everywhere. In this case, our system of equations simplifies to include only zonal wind and geopotential, and we can quickly find that $\omega = \pm kc$. However, the negative solution cannot be accepted because it violates our boundary conditions that require a solution that decays with latitude. Therefore, the dispersion relation of this wave is $\omega = kc$.

Slide 15: This means that the wave is nondispersive; its group velocity and phase speed match. It is essentially an easterly propagating Poincaré wave, but since f = 0 at the equator, it is basically a large-scale, relatively slow-moving equatorial gravity wave. The wave flow consists of zonal wind anomalies only, with convection occurring there is convergence. We say in this case that the convection is in quadrature with the geopotential because the phases of the two are shifted by a quarter-wavelength.

Slide 16: Plotted on a single dispersion diagram, we can see where the various waves fall in frequency-wavenumber space. The lines each represent a different solution for various values of *n*. Note that the Kelvin wave solution satisfies our general dispersion relationship for *n* = -1, so it is listed as so on this plot. Consistent with the math described, the equatorial Rossby waves are low frequency disturbances, with a frequency that asymptotes to the same value for all *n* at large *k*. Inertio-gravity modes are seen at the top, with eastward propagating disturbances to the right of the vertical frequency axis. The Kelvin wave is the only mode that is a straight line. If we look carefully, we will see that omega divided by k is the same as the slope of this line, indicating that the wave is non-dispersive. The mixed Rossby-gravity waves are continuous with the *n* = 0 eastward propagating inertio-gravity wave.

Slide 17: The various waves can be tracked in space by monitoring OLR from space. This example of a Hovmöller plot tracking OLR anomalies is averaged within 5 degrees of the equator. Time increases moving down, so that any contiguous regions of the same color moving down and right in the figure are moving eastward with time, and any features moving down and to the left are moving westward in time. Kelvin waves are highlighted by blue shapes, and equatorial Rossby waves are represented by red circled areas. Dashed lines represent the negative, convectively inactive, phase of the wave. We will discuss the Madden-Julian Oscillation, or MJO, denoted by the black lines, in a future module.

Slide 18: Some of the waves are also evident in the zonal wind, especially in the upper troposphere. Over the East Pacific, where convection is not supported at the equator because of low SSTs, the OLR signal of many of the Kelvin waves does not persist, but the anomalous zonal flow signature can be tracked. Several Rossby waves are also seen throughout the tropics in this example.

Slide 19: In contrast, waves are more difficult to track using the lower-tropospheric zonal wind because—to obey mass continuity—the magnitude of the anomalous zonal flow in the wave is lower.

Slide 20: We can also examine Hovmöller diagrams that are not averaged on the equator. This plot represents the mean value of 200 mb meridional wind within the latitude band of 10°–20°N as a function of longitude and time. Several westward propagating disturbances are seen here with high frequency. Only a few project onto the equatorial Rossby wave solution. Most of the waves seen in this plot are related to westward propagating inertio-gravity waves. We will continue to look at diagrams similar to these as we move forward through the lecture series, occasionally adding other variables such as velocity potential, which essentially smooths out horizontal variations in the flow. This will be useful in particular for tracking Kelvin waves and disturbances like the MJO.

Module 5.2

Slide 1: In this module, we will consider that the free wave solutions from the previous module are actually coupled to diabatic heating in the tropics. We will then examine the vertical structure typically seen in these equatorial waves coupled to diabatic heating.

Slide 2: The actual pattern of OLR in equatorial waves projects well onto Matsuno's solutions for equatorial waves, which are plotted in these two panels by lines that represent different phase speeds for a given value of *n*. The waves with convection that is asymmetric about the equator include the even-numbered inertio-gravity wave, the n=0 eastward inertio-gravity mode, and the mixed Rossby-gravity wave. The waves that contain convection that are symmetric about the equator include the odd-numbered westward inertio-gravity waves, the equatorial Rossby wave, and the Kelvin wave. The colors indicate pairs of frequencies and wavenumbers at which variability in observed tropical OLR occurs between 15°N and 15°S. However, the observed phase speeds are generally less than what would be predicted by dry equatorial wave theory—meaning the theory without considering the effects of moisture and diabatic heating on the circulation, which must be considered. In other words, when we consider that these waves have vertical structure and do not actually occur in a single-layer shallow fluid, the thermodynamic equation must be considered; it links the anomalous flow to the diabatic heating through mass continuity.

Slide 3: In the previous module, we defined a Rossby radius, which describes the off-equatorial distance at which an equatorial gravity wave begins to feel the effects of rotational flow. Here it is shown as dimensional, with units of meters. This distance depends on a quantity we will call the equivalent depth. This quantity is related to the stability of the atmosphere and the vertical wavelength of diabatic forcing. Equivalent depth corresponds to the depth that a shallow water gravity wave would need to give a phase speed of root-g-h that matches the observed phase speed in nature. It is only a theoretical depth in our shallow water model of equatorial waves and does not correspond to any actual depth in nature. We can readily examine how convective coupling might occur when heating is first baroclinic, which you may recall means that the heating peaks in the middle troposphere, is the same sign throughout the troposphere, and goes to zero at the ground and the tropopause. For a typical environmental lapse rate of 7°C/km, and a tropospheric depth of about 14 km—corresponding to a vertical wavelength for heating of 28 km—the equivalent depth would be about 200 m, which is denoted by the red arrow. A 200 m equivalent depth corresponds to a phase speed of about 45 m s^{-1}.

Slide 4: We can combine our horizontal momentum equations, the mass continuity equation including vertical motion, and the thermodynamic equation to get an expression for how geopotential changes as a function of divergence in the wave. The vertical gradient of geopotential—or the thickness—is proportional to the temperature, and no diabatic heating is included in the thermodynamic equations. The resulting equation in the red box looks essentially the same as the mass continuity equation written in terms of geopotential, except that the equivalent depth is included and the phase speed is root-g times equivalent depth. Here we have assumed that geopotential has a first baroclinic vertical structure.

Slide 5: The solution we just outlined was for dry equatorial waves with an assumed vertical structure. "Dry" means that the waves are uncoupled to latent heating; these are also "free" waves. Some dry Kelvin waves, for example, are observed in nature; however, the power in the Kelvin wave part of the wavenumber-frequency diagram is concentrated along lines of equivalent depth that correspond to slower moving waves. For example, the observed propagation of equatorial Kelvin, or eastward moving gravity, waves correspond to an equivalent depth of about 25 m. We can try to emulate this using our existing mathematical framework by simply adding diabatic heating or cooling Q to the thermodynamic equation. We now have a set of equations with no analytical solution. One way of simplifying the equation system is to assume that the diabatic heating varies linearly with the vertical motion, such that we can express Q in terms of ω. This is basically another way of saying that the atmosphere and convection are in quasi-equilibrium, which is a topic for a more advanced course in tropical meteorology than this one at NPS. In this case, the so-called "effective static stability" is reduced, such that the value of N in the expression for equivalent depth is reduced. For this, the phase speed of the coupled wave is reduced proportional to the offset between adiabatic cooling and condensational heating associated with upward motion. The same is true for opposite sign in downward motion, except that radiative cooling becomes the dominant diabatic term. Observations do indicate that this approximation appears to be largely true in the tropics.

Slide 6: Another possible explanation for why equatorial waves propagate slower than the "free" solutions is that the propagation speed is a combination of the phase speeds of both the first and higher baroclinic modes. Indeed, diabatic heating—especially in mature mesoscale convection—takes on a second baroclinic vertical structure that cannot be captured by the system of equations presented that assumes that all variables have a first baroclinic structure. The second baroclinic mode has a propagation speed of only 23 m/s, compared to speed of 49 m/s for the first baroclinic mode in an approximately 16 km deep troposphere. This is related to the moisture-stratiform instability, which is one way of describing the growth of equatorial modes. Again, detailed explanation of this is saved for a more advanced discussion. For now, we'll settle with the notion that a complete understanding of mechanisms that explain equatorial wave propagation remains unobtained.

Slide 7: The vertical structure of the equatorial waves is often tilted with height, and these features can also help identify waves. This and following plots are time-height diagrams, for which the zero-time lag on the x-axis represents the time at which precipitation in the coupled wave is at its maximum at some location. A positive time lag represents the vertical wave structure above the same location at some time after the precipitation maximum. We will first start by looking at the typical Kelvin wave vertical structure. Dark shading represents positive anomalies. For an eastward propagating disturbance like a Kelvin wave, a negative time lag is experienced at a location east of the convective maximum. Therefore, the x-axis on these plots are reversed, with negative lags on the right. This way, the x-axis can also be thought of as a longitudinal axis from west on the left to east on the right. The zonal wind structure of the Kelvin wave, therefore, tilts westward with height. Convergence is seen at low-levels, and divergence is found aloft where convection occurs. The convection is accompanied by a negative temperature anomaly in the lower-troposphere and a positive temperature anomaly aloft. The specific humidity anomaly tilts westward with height as well. This means that moisture in the low troposphere is at its largest concentration before the convective maximum in the wave occurs. The moisture is then transported upward via convection and is at its maximum in the upper troposphere around the time of the convective maximum and shortly after, when decaying stratiform precipitation may be present. Thus, convection in a Kelvin wave acts to transport MSE-rich air from the lower troposphere to the upper troposphere, making the atmosphere more stable when it is unstable, and making it less stable when it is already stable.

Slide 8: The equatorial Rossby wave also has meridional wind structure. The anomalies plotted here are at an off-equatorial site around 7°N. Note how the x-axis time-lag is plotted for a westward propagating disturbance. Again, the left sides of the plots represent west in space. Zonal and meridional wind anomalies tilt westward with height. The temperature anomaly shows a higher order baroclinic structure, and the humidity anomalies are in phase with the convection.

Slide 9: The mixed Rossby-gravity wave exhibits meridional winds that tilt eastward with height near 7°N in the troposphere. In the stratosphere, they tilt westward with height. The vertical structure of temperature is similar, with a second baroclinic structure in the troposphere present. Like the Kelvin wave, the humidity anomalies in the lower troposphere precede the convection, and the upper tropospheric humidity anomaly is nearly in phase with convection.

Slide 10: The much faster moving n = 1 westward propagating inertio-gravity wave near the equator has a vertical structure at the equator that looks like the off-equatorial mixed Rossby-gravity wave structure.

Slide 11: The $n = 0$ eastward propagating inertio-gravity wave has a vertical structure at 7°N as shown. Lower troposphere positive humidity and temperature anomalies again lead the convection, and a westward tilt with height is seen for both fields, except that in the upper troposphere and stratosphere, the temperature anomaly tilts eastward with height. Meridional and zonal wind anomalies tilt westward with height.

Slide 12: All the waves discussed generally have very different spatio-temporal scales. For example, observed inertio-gravity waves tend to have short wavelengths and propagate quickly. Equatorial Rossby, Kelvin, and $n = 0$ waves typically have lower wavenumber and larger wavelength. The temporal scale of variability in disturbances that propagate slowly and with long wavelength tends to be shorter than that seen in inertio-gravity waves. However, all the waves that possess at least some gravity wave characteristics share the same transition of convection from shallow clouds to deep cumulonimbi. In the suppressed phase of the wave, dry air subsides, creating an inversion that suppresses convection that is rooted in the boundary layer. As the upward branch of the wave approaches, the free troposphere becomes less statically stable, and convection can grow. The convection deposits moisture into a sequentially deeper layer, allowing subsequent convection to grow even taller because the air it entrains is less dry than that entrained by the previous convection. Eventually, the atmosphere becomes unstable and moist enough to support deep convection. This is why we often see a build-up in moisture, such that the vertical structure of humidity anomalies tilts opposite the direction of motion of a wave with height. This paradigm of convective build-up in gravity wave disturbances, or those that contain gravity wave components, is applicable across a wide range of spatial and temporal scales in the tropics. Next, we will apply it to an even slower moving convectively coupled wave disturbance, that associated with the Madden-Julian Oscillation.

Module 5.3

Slide 1: This module will discuss some of the basic features of the Madden-Julian Oscillation, or MJO. The MJO is the dominant variability in precipitation and many other atmospheric and upper ocean variables on time scales of 20–100 days, also known as sub-seasonal or intraseasonal variability. Although it has noticeable impact on weather around the globe, it is still not well represented in most numerous models of the atmosphere and ocean, and the underlying dynamics that control the MJO are still unexplained.

Slide 2: What became known as the MJO was first noticed in the 1960s but was popularized by a set of papers by Madden and Julian in the early 1970s. They documented distinct minima and maxima in surface pressure at various ground stations in the tropics every 40 to 50 days, such as in the top plot, which illustrates a raw time series of pressure at top and a 45-day band-pass filtered pressure timeseries at bottom from the West Pacific. Rawinsondes as the payload on weather balloons captured similar variability in zonal wind at various altitudes. By running a band-pass filter on time series of various variables, they were able to demonstrate a statistically significant variability in the tropical atmosphere at the sub-seasonal time range. An example of band-pass filtered zonal wind and surface pressure over a 4-year period is shown at bottom. High amplitude variability corresponds with a strong MJO, while other periods of time show little variability in the sub-seasonal band. Since these early papers, the MJO has been extensively studied and linked to variability in weather events all over the world; however, a complete explanation for what drives the MJO is still unknown.

Slide 3: Timeseries of precipitation with similar variability also occurs, indicating that the MJO is likely convectively coupled. Shown here is a time series of radar-derived precipitation from the central Indian Ocean, created from data collected during the DYNAMO field campaign in 2011, a multi-national, multi-agency field experiment aimed at better understanding the initiation processes for convection in the MJO. Over the 3.5-month long period plotted here, three broad peaks in precipitation are denoted, representing three active peaks of convection in the MJO, each separated by a minimum in convective activity. In this example, the broad peaks are separated by about 30 days. Superimposed on the MJO signal are several high frequency rainfall events, denoted by the narrower spikes in the rainfall time series. These were associated with other equatorial waves—especially Kelvin, westward propagating inertio-gravity, and equatorial Rossby waves. Thus, a variety of different wave modes exist simultaneously at various frequencies and can constructively or destructively interfere with one another. For example, if the downward branch of an inertio-gravity wave is present at the same time as the upward branch of an MJO, little precipitation may occur where the inertio-gravity wave suppresses convection. However, because the frequency of inertio-gravity waves is much higher than that of the MJO, as we might recall from wavenumber-frequency power spectra of tropical outgoing longwave radiation, the rainfall recovers rather quickly after a minimum.

Slide 4: The MJO manifests itself primarily as variability in convection over the tropical Indo-Pacific warm pool, roughly within the orange ellipse shown here. Convection in the MJO is often first observed in the left part of the ellipse over the Indian Ocean. A primary characteristic is its low frequency, with maxima in rainfall occurring only approximately once per month or less. Another defining feature of the large-scale convective region during an active MJO is that it moves eastward at speeds of, on average, only about 5 m/s. This is much slower than the propagation speeds of free Matsuno equatorial modes. The MJO impacts weather around the globe, and a few examples are highlighted here. For example, sub-seasonal variability in surface temperatures in the Arctic have been linked to MJO activity in the tropics. Rainfall and severe weather events, such as atmospheric river events, Santa Ana wind events, or severe weather outbreaks in the central U.S. have been linked to the MJO. The MJO modulates monsoon activity as well as tropical cyclone activity in regions where TCs occur. Many other examples have been documented in scientific literature and are not shown here, but I could probably color in just about every region on Earth with some impact that the MJO has on regional weather on sub-seasonal time scales.

Slide 5: The basic circulation pattern of an MJO when convectively active is closely described by the Gill-type response to equatorial heating. The response to symmetric latent heat release centered on the equator is a combined Kelvin and Rossby wave structure. Convergence is seen on the equator near the convection, while two off-equatorial gyres develop slightly west of the longitude of the convective maximum. This pattern tends to become coupled to convection as it moves toward the east; however, the circulation becomes more complicated for cases when the maximum latent heating is not symmetric about the equator, which is especially common during boreal summer, when the MJO and the South Asian summer monsoon interact to form what some call the Boreal Summer Intraseasonal Oscillation.

Slide 6: The cloud pattern associated with MJO variability sometimes resembles the circulation response to Gill-type heating. In this figure, the equator is located along the horizontal white line between the islands labeled Male and Gan.

Slide 7: One can somewhat envision how the cloud population is focused along the equator ahead of the area of maximum latent heating, shaded in red. The clouds to the north and south of the heating maximum are contained within gyres and even show some sign of positive relative vorticity. To the immediate west of the latent heating maximum is an area with little convection. This is reinforced by the equatorial flow of drier air from higher latitudes around the Rossby gyres. Other waves are superimposed onto this coupled Kelvin-Rossby wave response to heating. For example, a relative void in convection is located immediately east of the red ellipse, but farther to the east is another region of active convection. This may represent something like an inertio-gravity wave superimposed onto the larger-scale, more slowly varying MJO circulation.

Slide 8: Because of its impacts on weather globally, great interest is taken in objectively identifying and tracking the upward branch of the MJO. A common way of doing so uses principal component analysis and projects horizontal fields of outgoing longwave radiation and/or zonal wind at various pressure levels onto the first two empirical orthogonal functions, or EOFs, that describe about 25% of the variance associated with either field. EOFs 1 and 2 respectively correspond to principal components, or PCs, 1 and 2. The methods create an MJO index that can be used to describe the magnitude of the MJO signal and the location of its upward branch. The first two EOFs can be illustrated as spatial patterns that look like what is shown on this slide, with the first two EOFs for two commonly used MJO indices, the real-time multivariate MJO index, or RMM; and the OLR MJO index, or OMI, shown here. The blue shading indicates where convection is located. One EOF has convection centered over the Maritime Continent, with regions of suppressed convection located to the east. The other significant EOF contains convection near the equator but is centered more over the West Pacific, with suppressed convection in the Indian Ocean. While the OMI uses only OLR, the RMM index uses both zonal wind and OLR. How the real OLR or zonal wind field projects onto these two EOFs determines the value of each index.

Slide 9: Such methods of identifying the MJO separate the MJO into 8 phases, which simply describe all potential combinations of the signs and relative magnitudes of the first two principal components. The eight phases make a cycle that is often seen to repeat itself. As an example, a positive PC1 and positive PC2 would indicate that convection is located over the Maritime Continent, where the blue colors were seen on the left of the previous slide. As shown in the top left, having positive values for both PCs means that we are in Phase 5 and 6 of the cycle. Negative magnitudes for both PCs would tell us that convection is particularly suppressed in those blue regions, and the RMM index would be in Phases 1 or 2, with the MJO upward branch—and perhaps convection—located more over Africa or the Indian Ocean. The figure on the right is called a phase space diagram. It is a common way of assessing the magnitude and location of the upward branch of an MJO. On the x-axis is the value of PC1, and on the y-axis is the value of PC2. The axis labels denote the phases the correspond with each pie slice of the phase space, and the approximate location within the tropics of the MJO upward branch is denoted within parentheses. The circle at the center of the plot denotes a magnitude of 1. Generally, the index must have a magnitude of at least 1 for a strong MJO to be considered active. Note that the magnitude of the index is simply the square root of the sum of the squares of the leading two PCs. These plots illustrate a timeseries of RMM (or other index depending on what is plotted) magnitude. For example, the red dot might be collected at some point in time by projecting the real OLR and zonal wind horizontal structure onto the leading two EOF structures. At the red dot, I have a large negative magnitude for PC2, meaning that the convective pattern is opposite that seen on the right-hand side of the previous slide, and a positive PC1, meaning that the spatial pattern of convection projects well onto the spatial pattern seen on the left-hand side of the previous slide. Based on this, we could conclude that the upward branch of the MJO is located over the Indian Ocean. The different colored lines in this example show daily values of the index plotted over the course of three months, with each month denoted by a different color. Because the MJO propagates toward the east, the line generally moves counterclockwise in the phase space displayed. You can see how the magnitude of the MJO signal varies with time as well. The farther from the center of the circle a point is, the stronger the MJO signal.

Slide 10: The typical anomalous OLR structure seen when the RMM is in each phase is shown here, with the green and blue shading indicating regions of anomalously positive precipitation. At the top is Phase 2, which features anomalous rainfall over the central Indian Ocean. As the phase progresses, the rainfall moves toward the east, and by Phase 7, the convection becomes partially decoupled from the circulation as the circulation moves over regions where convection is not well supported by the environment, such as in the downward branch of the Walker circulation, where the atmosphere is stable and SSTs are low. Any convective signal that persists generally moves rapidly through the Western Hemisphere in Phases 7 through 1 before potentially repeating when the upward branch of the wave reaches the Indo-Pacific warm pool again. Note again that these are anomalous rainfall signals associated with the MJO. Other variability is always superimposed on top of the MJO signal. Indices such as RMM and OMI try to find the component of the total signal that looks like these spatial patterns.

Slide 11: The same example shown before is seen at left in this slide. If we start following the line where the word “START” is seen in the top right quadrant, and follow the line counterclockwise, we see one active period over the Indian Ocean along the blue line in October, then following the line around, another active period in November, and continuing to follow the line around, an apparent stalling of the MJO during December over the Maritime Continent. On the right is a Hovmöller time series of satellite-derived rainfall with locations in the tropics denoted along the top axis. Three broad regions of enhanced precipitation are seen to propagate toward the east; each were associated with an upward MJO branch. In fact, a circulation signal associated with the MJO moved all the way around the world between the November and December events depicted here; however, the RMM index essentially got stuck over the Maritime Continent as convection persisted there. This is an example of one limitation of objective algorithms that are based on PCA and projecting fields onto the first two EOFs. They don’t necessarily portray the MJO signal correctly when the convection is weak. This can happen sometimes when a circulation signal is present over the East Pacific but no convection is present because equatorial SSTs there are too low to support convection.

Slide 12: Some other indices are used, and a list is shown here. If interested, you can take a look online for details about these other indices, which use OLR or velocity potential in various ways but still project fields onto the leading two PCs of sub-seasonal variability. One of these indices is the Velocity Potential MJO, or VPM, index. It replaces OLR with velocity potential.

Slide 13: The velocity potential is primarily concerned with the irrotational component of the large-scale flow, so inherently, it fundamentally tries to track any potentially existing gravity wave component of equatorial disturbances that propagate eastward. The velocity potential is essentially a smoothed divergence field such that negative velocity potential represents divergence. Velocity potential is defined such that its gradient describes the divergent component of the wind. The Laplacian of the velocity potential is zero for incompressible flow.

Slide 14: Based on their mathematical definition, velocity potentials smooth out areas of divergent flow. It is often tracked at 200 hPa as a diagnostic of where the upward branch of equatorial waves are typically present. In this Hovmöller diagram, brown areas represent anomalous large-scale convergence at 200 hPa—near the tropopause, which would correspond via mass continuity to anomalous downward motion beneath 200 hPa. Green areas denote large-scale divergence aloft, which is consistent with anomalous upward motion beneath.

These velocity potential anomalies are easily seen to move eastward throughout the tropics at times, and their circumnavigation is often apparent.

Slide 15: Anomalies of velocity potential can also be tracked on plan view maps, with forecasts into the future projecting where negative velocity potential anomalies may exist. This can provide some crude medium-range predictability of generally what part of the tropics convection may be more active, and many times, negative velocity potential anomalies in the upper troposphere correspond with regions of active MJO convection. In this example, a region of anomalous divergence aloft is seen to be present over the Indian Ocean and forecasted to move eastward toward the Maritime Continent over the following two weeks. Forecasters interested in tropical cyclone activity also pay attention to the propagation of velocity potential anomalies, because they may provide some insight, for example, about the potential for increased tropical cyclone activity as a region of anomalous tropospheric upward motion approaches.

Module 5.4

Slide 1: This module briefly overviews the dynamics of the initiation and propagation of the Madden-Julian Oscillation. A full treatment of this topic if far more than we can discuss in just one lecture; however, some major points are covered in this video.

Slide 2: The fundamental features of convection associated with the MJO are listed here. The MJO is large in spatial scale. The circulation anomaly of the MJO projects strongly onto wavenumbers 1 and 2. The convection in the MJO propagates only eastward and with a mean motion of about 5 m/s, although the circulation signal can move much faster when not strongly coupled to convection over the Indo-Pacific warm pool. The periodicity falls within the sub-seasonal time range, but the MJO is irregular, with precipitation peaks occurring at various intervals. Sometimes, more MJO convective events more evenly spaced in time are clearly linked to a prior event; however, others are not. This is part of the discussion we will have during our paper discussion for this week.

Slide 3: A major open question concerns how convection in the MJO develops in the first place. We know from many prior modules that the depth of convection is sensitive to moisture concentration in the lower free troposphere. If dry air is entrained into buoyant updrafts, the updrafts are diluted and eventually lose all positive buoyancy. However, convection is a major process through which moisture is transported out of the boundary layer and into the lower free troposphere. In the MJO, moisture is observed to increase in the troposphere in concert with a deepening of convection as shown in the figure below. When averaged over many MJO events, this slow build-up of moisture and convection can occur over a 2–3-week long period, although observations indicate that the process often happens much more quickly. One major question involves how the moisture gets into the free troposphere. Does it get horizontally advected there, thus allowing convection to then develop, or does it get transported by clouds upward, allowing for subsequent clouds to grow deeper and transport moisture to even higher levels?

Slide 4: The progression of moisture anomalies through height with a passing MJO is illustrated by the plot here, which is similar to the vertical structure plots we saw for other convectively coupled equatorial waves in a previous module. Dark shading in this plot indicates positive anomalies of specific humidity associated with the MJO. The anomalous OLR in the MJO is shown at the top. Note that the x-axis here is shown as longitude. The largest positive anomalies of humidity are located in the middle to upper troposphere at the same time the OLR anomaly is at its minimum. However, the largest specific humidity anomaly in the lower troposphere is located to the east of the OLR minimum. This is another example of how the humidity field is tilted with height in an equatorial wave, and the tilt does have some similarity to an eastward propagating gravity wave, where shallow convection is coincident with positive moisture anomalies ahead of the maximum upward motion anomaly, while deep convection and stratiform precipitation is prevalent where the moisture anomalies extend into the upper troposphere. For an eastward propagating disturbance like convection in the MJO, this figure suggests that the humidity anomalies are first experienced in the lower troposphere then aloft.

Slide 5: Additional evidence for this can be seen in timeseries of vertical profiles of relative humidity derived from rawinsonde data. Green areas on this plot denote times and pressure levels where humidity was high, while red indicates dryness. Each row represents a timeseries at a different location in the Indian Ocean. This set of timeseries spans two MJO events that started in mid-October and mid-November. Prior to both, a dry troposphere tops a moist boundary layer. At some point, high values of relative humidity are suddenly present up to about 500 hPa, around the level of a climatological region of stability in the tropics. After a few days, convection was able to penetrate this layer and deposit moisture in the upper troposphere as well. At this point would we say that the MJO was convectively active. This stepwise progression of moisture strongly indicates a close linkage between clouds and moisture, a relationship that is further examined in our paper discussion for the week.

Slide 6: Radar suggests that convection leads the moistening of the troposphere. By analyzing moisture at various levels from rawinsonde data, and the presence of convective and stratiform radar echo, we are able to lag-correlate deep convection and stratiform with tropospheric moisture. In this table, the decimal number indicate a maximum lag correlation between the variables listed on the left. The number in the parentheses indicates the times at which the lag-correlations were highest, and a positive number indicates that the variable in the first column precedes the variable in the second column. The different columns under smoothing interval just show results when the timeseries are smoothed at various levels. Just focusing on the third column, however, we can see that at spatial scales of a radar, convection and 850 mb humidity are nearly coincident with each other. 300 mb humidity lags the convection by about 9 hours, however. Stratiform precipitation occurs after convection and is more coincident with the humidity field between 700 and 500 mb. This type of analysis strongly supports the notion that the convection *leads* to the humidity—at least on short time scales and small spatial scales. There is scale similarity between this process and the relationships between these variables in the MJO, although the lag periods are longer. Combined with other analysis not shown here, this points to a strong contribution of clouds in moistening the environment sufficiently to allow deep convection in the MJO to become established. This is required before the circulation in the MJO can become fully coupled with diabatic heating in the deep convection. In the paper discussion, we will discuss the potential role of circumnavigating waves in allowing the growth of shallow convection and subsequent moistening of the lower troposphere; however, any process that leads to the large-scale moistening of the lower troposphere over a large region of warm SSTs where convection can be sustained for many days could potentially kick off a strong MJO event.

Slide 7: What we just reviewed was related to the process of MJO initiation; that is the transition from a period of convectively suppressed conditions over the Indo-Pacific to an active period. A problem that has, as of yet, been mostly treated separately is the maintenance and propagation of an existing MJO signal once convection in the MJO has been established. Taking a look again at the wavenumber-frequency power spectrum of tropical OLR, we recall that a peak in power existed at very low frequency and at mostly small wavenumbers, with a peak at wavenumbers 1 and 2. This partially aligns with Kelvin wave but is distinct (although with consideration of mean westerlies can be shown to be continuous with the Kelvin wave part of the spectrum, a topic for a more advanced discussion). Next, we will review one possible theory that explains MJO propagation. It is known as moisture mode theory. It begins with the assumption that weak temperature gradients exist in the tropics and uses moisture only as the prognostic variable. Unlike the dry Matusno modes we discussed previously, the moisture mode theory incorporates moisture and describes MJO growth and propagation as being dependent upon the interaction between moisture and the MJO circulation.

Slide 8: While we will not discuss this in great detail, the dispersion relationship for the MJO when making such assumptions is seen at the top right. It is dependent upon the horizontal wind field, distribution of moisture, and the gross moist stability, which is the last term in the dispersion relation. The gross moist stability is described as being dependent upon a term that is related to the feedback between clouds and radiation. Essentially, the presence of upper level clouds that develop in moist environments increases tropospheric radiative heating because less longwave radiation escapes to space. The additional diabatic heating supports additional vertical motion which transports even more moisture to the upper troposphere. This is part of the growth mechanism of the moisture wave, which we can see in the imaginary component of the dispersion relation. The dispersion relationship is denoted by the curved lines that correspond with the power at low wavenumbers and frequencies as shown here.

Slide 9: The phase speed of the moisture wave is always toward the east, while the group velocity is toward the west. These plots show Hovmöller diagram of actual band-pass filtered OLR anomalies in the MJO, with time increasing going up on the plots. The green areas denote OLR minima, and they are propagating toward the east. However, the apparent energy in this mode moves toward the west, which is also clearly seen in this plot and denoted by the negatively sloped dashed lines.

Slide 10: A study of successive MJO events—meaning those that are immediately preceded by another MJO event—shows that the MJO propagates over the Indo-Pacific at the theoretically derived phase speed but more like a convectively coupled Kelvin wave over the Western Hemisphere. The blue shaded regions in these plots indicate anomalies of column mean latent heating in an equatorial wave, shown again on a Hovmöller plot, now with time increasing going down. Several MJO events were composited together to create these plots. The x-axis spans the entire globe, while the y-axis spans about 60 days. The purple line outlines the track that the blue shaded region, which is essentially the upward branch of the MJO, would take if it propagated like a Kelvin wave coupled to diabatic heating anomalies associated with latent heat release and radiative cooling. The orange line shows the propagation of the MJO if it were a Kelvin wave where convection were not active but as a moisture wave where convection was active. Clearly, it agrees very closely with the actual track of the OLR anomaly shaded in blue.

Slide 11: However, numerous other theories exist that can also explain some of the salient features of the MJO that were described at the beginning of this module. All capture the eastward propagation of the MJO in some way, but the fundamental dynamics in each theory are different. One theory, the gravity wave theory, does not even require moisture to describe the eastward propagation. Each theory, including the moisture wave theory, has its own sets of assumptions and limitations such that no theory is widely agreed upon and accepted. A combined theory that explains both MJO initiation and propagation continues to be an active goal of tropical meteorology researchers.