

MR3252: Tropical Meteorology

Fundamentals of Moist Tropical Convection

Main Topics:

- Review of fundamental equations
- Factors influencing the growth of tropical convection
- Effect of moisture on convection

Shallow cumuli during DYNAMO
near Addu City, Maldives.

In mid-latitude dynamics, **quasi-geostrophic** theory is largely used to explain synoptic-scale motions.

Start with the basic primitive equations in isobaric coordinates, neglecting friction (from Holton, Chapter 3):

Horizontal
Momentum

$$\frac{D\mathbf{V}}{Dt} + f\mathbf{k} \times \mathbf{V} = -\nabla_p \phi$$

Mass
Continuity

$$\nabla \cdot \mathbf{u} = 0 \quad \longrightarrow \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0$$

Thermodynamic

$$\frac{D_h T}{Dt} - S_p \omega = \frac{J}{c_p} = Q$$

Key to QG theory is the assumption that, to first order, motions are in **geostrophic balance**.

Why does this not work in the Tropics?

$$\frac{D\mathbf{V}}{Dt} + f\mathbf{k} \times \mathbf{V} = -\frac{1}{\rho} \nabla_p \phi$$

Key to QG theory is the assumption that, to first order, motions are in **geostrophic balance**.

Why does this not work in the Tropics? $f \approx 0$.

Only for planetary length scales in the Rossby number small enough (e.g. $\ll 1$) to be able to assume geostrophic balance.

On spatial scales of $O(1000\text{km})$ there cannot be a balance between pressure gradient force and Coriolis in the Tropics.

All three terms above are similar enough in magnitude that none (including the Lagrangian of the wind vector) can be neglected.

Also consider the thermodynamic equation:

$$\frac{D_h T}{Dt} - S_p \omega = Q$$

In QG theory, diabatic heating (Q) can be ignored to estimate many large-scale mid-latitude motions. This leaves the time tendency of temperature affected by advection and vertical motion.

In the Tropics, moist convection is central to dynamics!

Moist heating drives vertical motion, but at the same time, an increase in upward motion (usually realized as a decrease in clear-air downward motion) can promote diabatic heating.

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = Q + S_p \omega$$

The feedback and interaction between convection and larger-scale tropical dynamics remains an open topic of research today.

What factors influence moist tropical convection? *Many, and the non-linear interactions between them are not fully understood!*

Leading factors impacting convection (**by altering buoyancy of convective updrafts**):

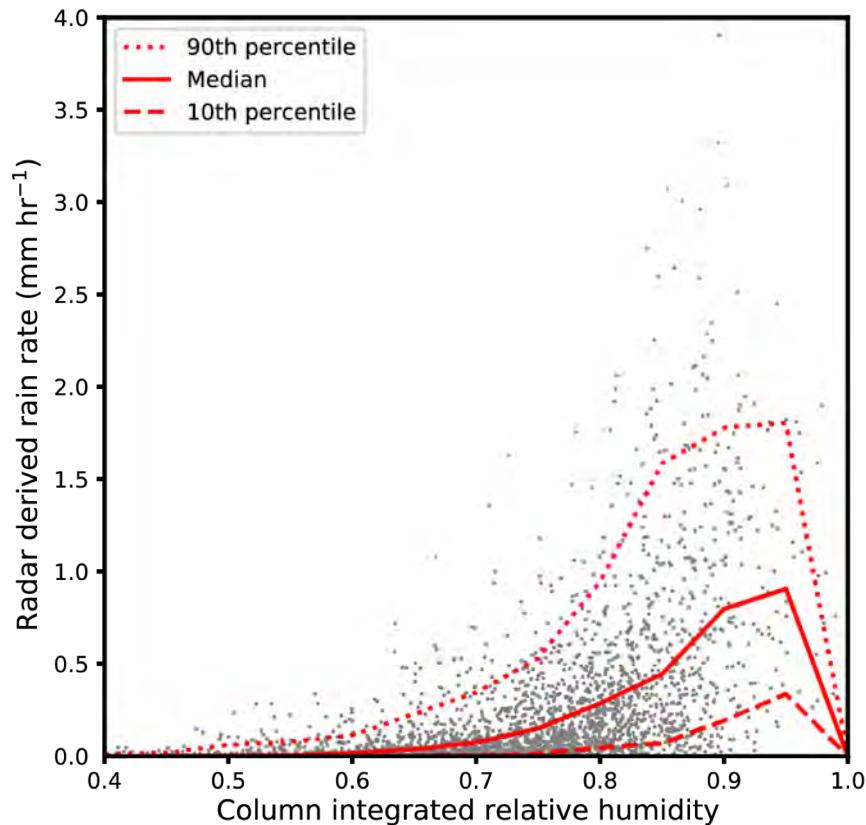
- 1) Moisture availability (A tautology to say that moist convection needs moisture, really. Of course, one needs moisture to support moist convection!)
- 2) Static stability
- 3) Surface fluxes
- 4) Wind shear
- 5) Low-level convergence (provides forcing)

However, these are impacted by other factors that are part of the large-scale dynamics that are themselves influenced by the convection:

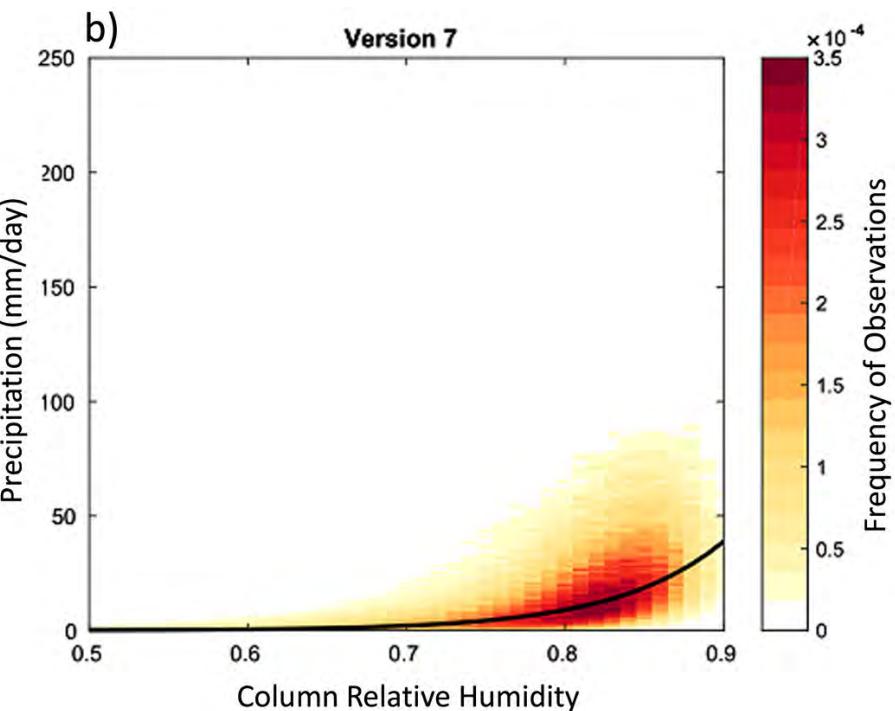
- Advection
- Near-surface wind
- Sea surface temperature
- Adiabatically driven vertical motion
- Vertical flux of moisture (i.e. moist convection itself)
- Low-level convergence

Moisture Availability

Radar Data (Powell 2019)



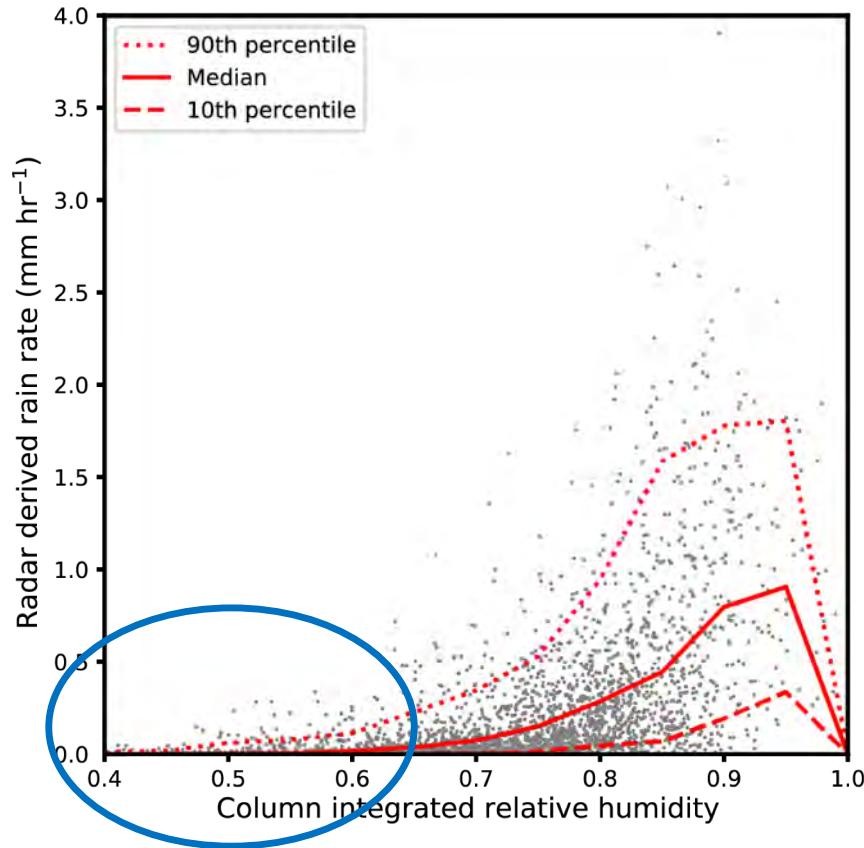
Satellite Data (Rushley et al. 2018)



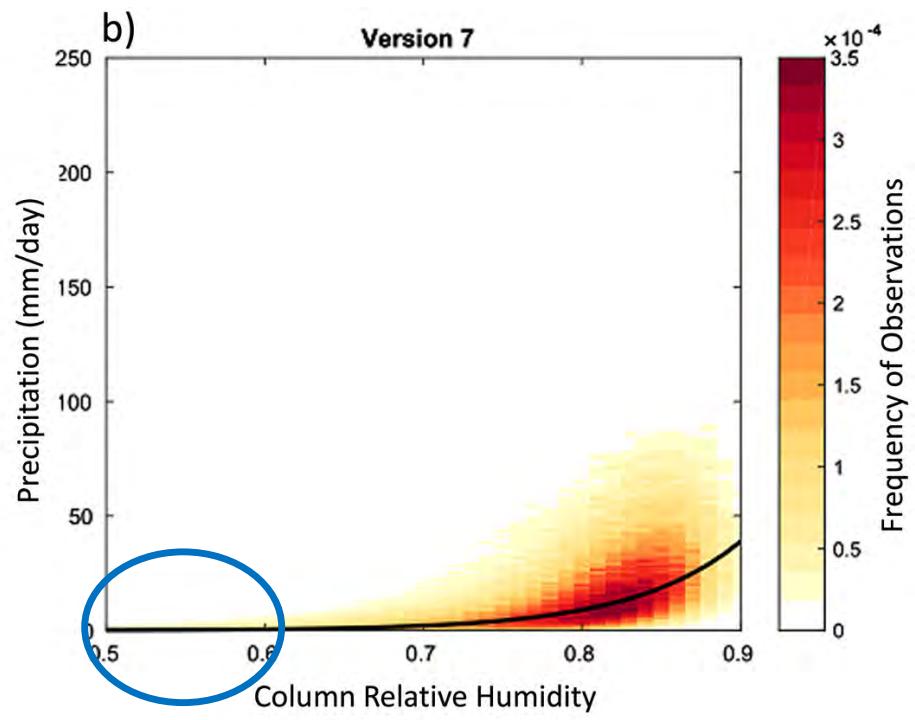
$$CRH = \frac{\int_{P_{sfc}}^{P_{top}} q \, dP}{\int_{P_{sfc}}^{P_{top}} q_{sat}(T) \, dP}$$

Moisture Availability

Radar Data (Powell 2019)



Satellite Data (Rushley et al. 2018)

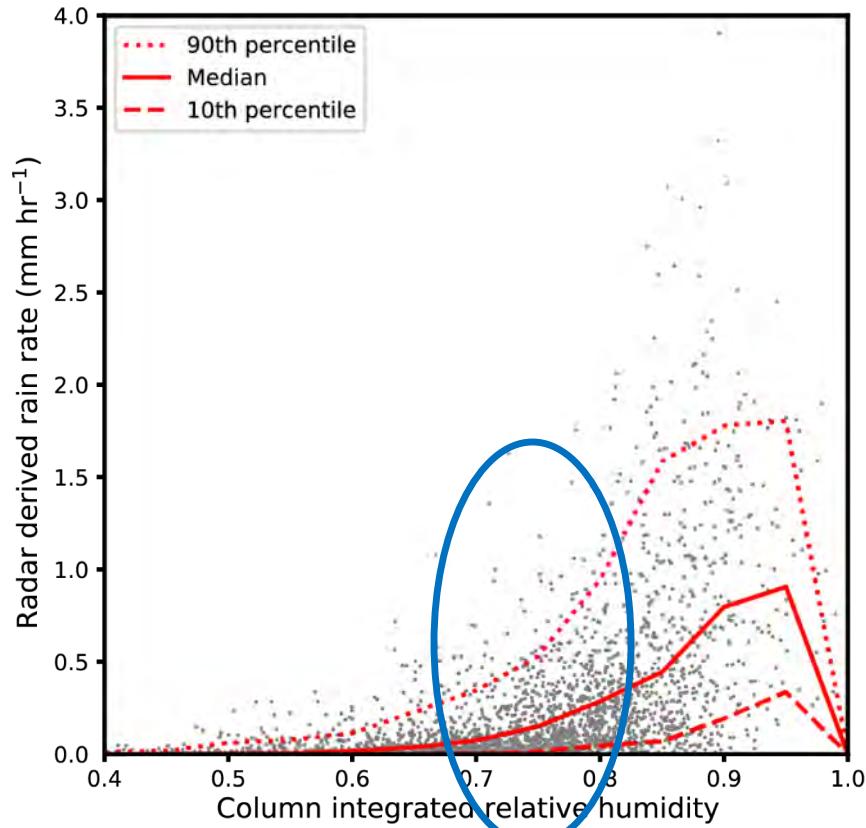


$$CRH = \frac{\int_{P_{sfc}}^{P_{top}} q \, dP}{\int_{P_{sfc}}^{P_{top}} q_{sat}(T) \, dP}$$

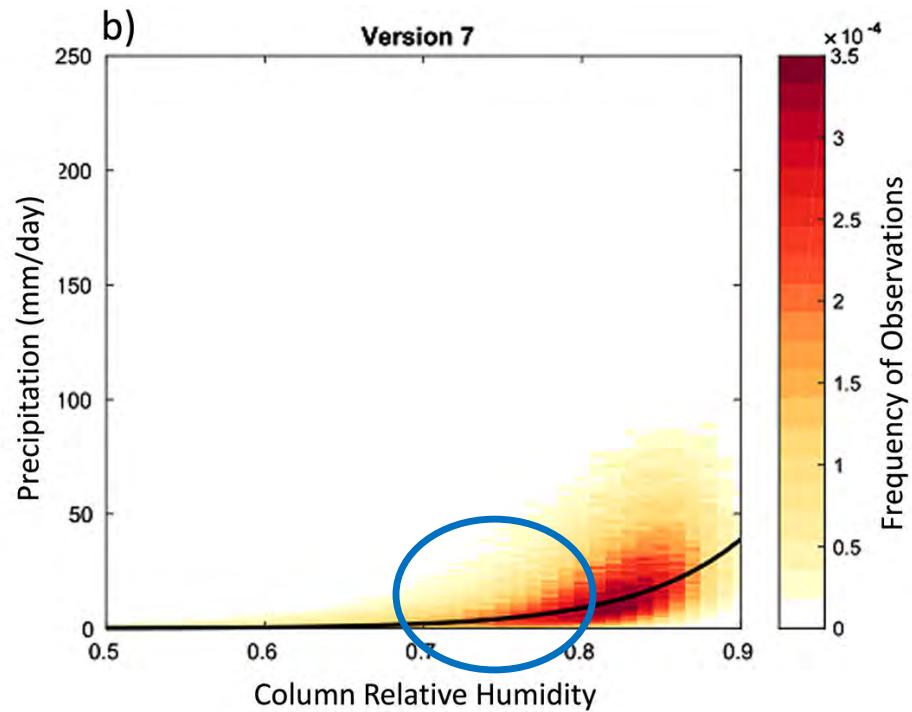
Low rain rates at low CRH.

Moisture Availability

Radar Data (Powell 2019)



Satellite Data (Rushley et al. 2018)

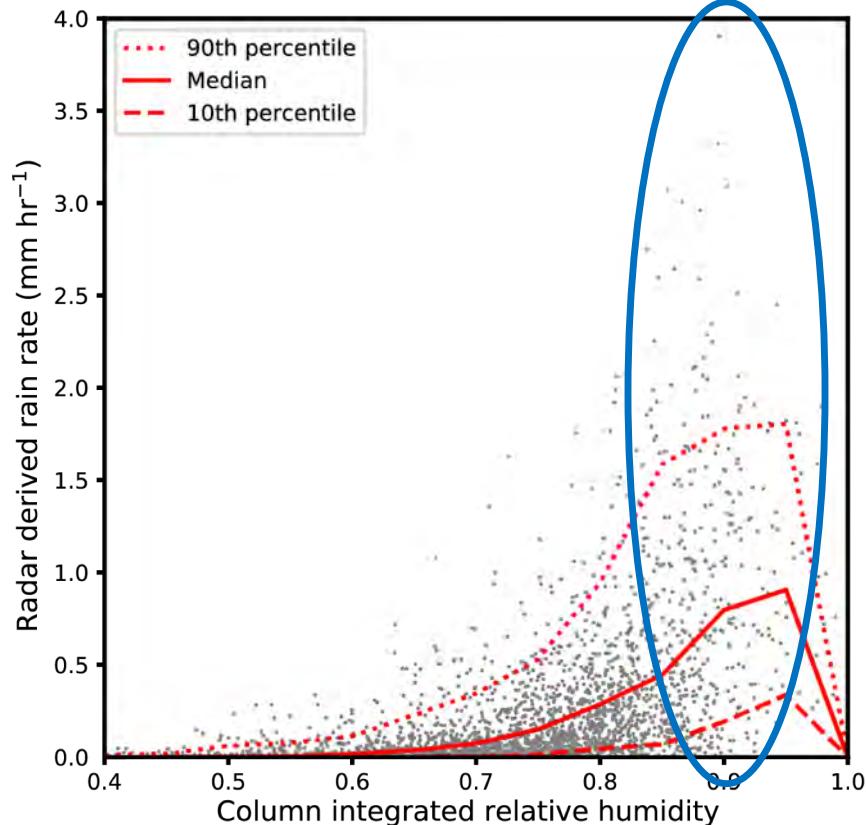


$$CRH = \frac{\int_{P_{sfc}}^{P_{top}} q \, dP}{\int_{P_{sfc}}^{P_{top}} q_{sat}(T) \, dP}$$

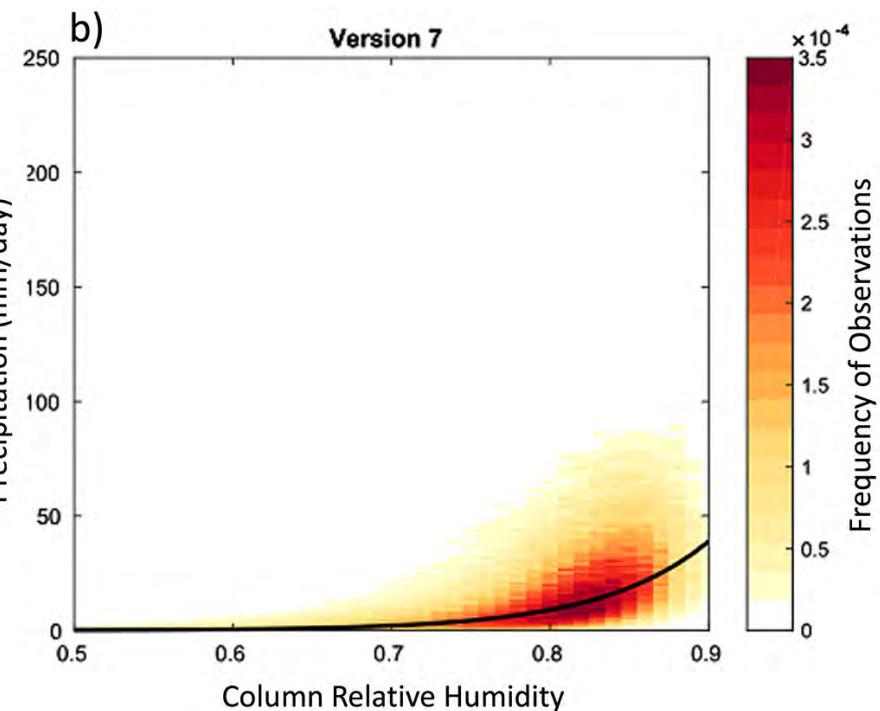
Exponential increase begins to increase quite rapidly between 70 and 80% CRH.

Moisture Availability

Radar Data (Powell 2019)

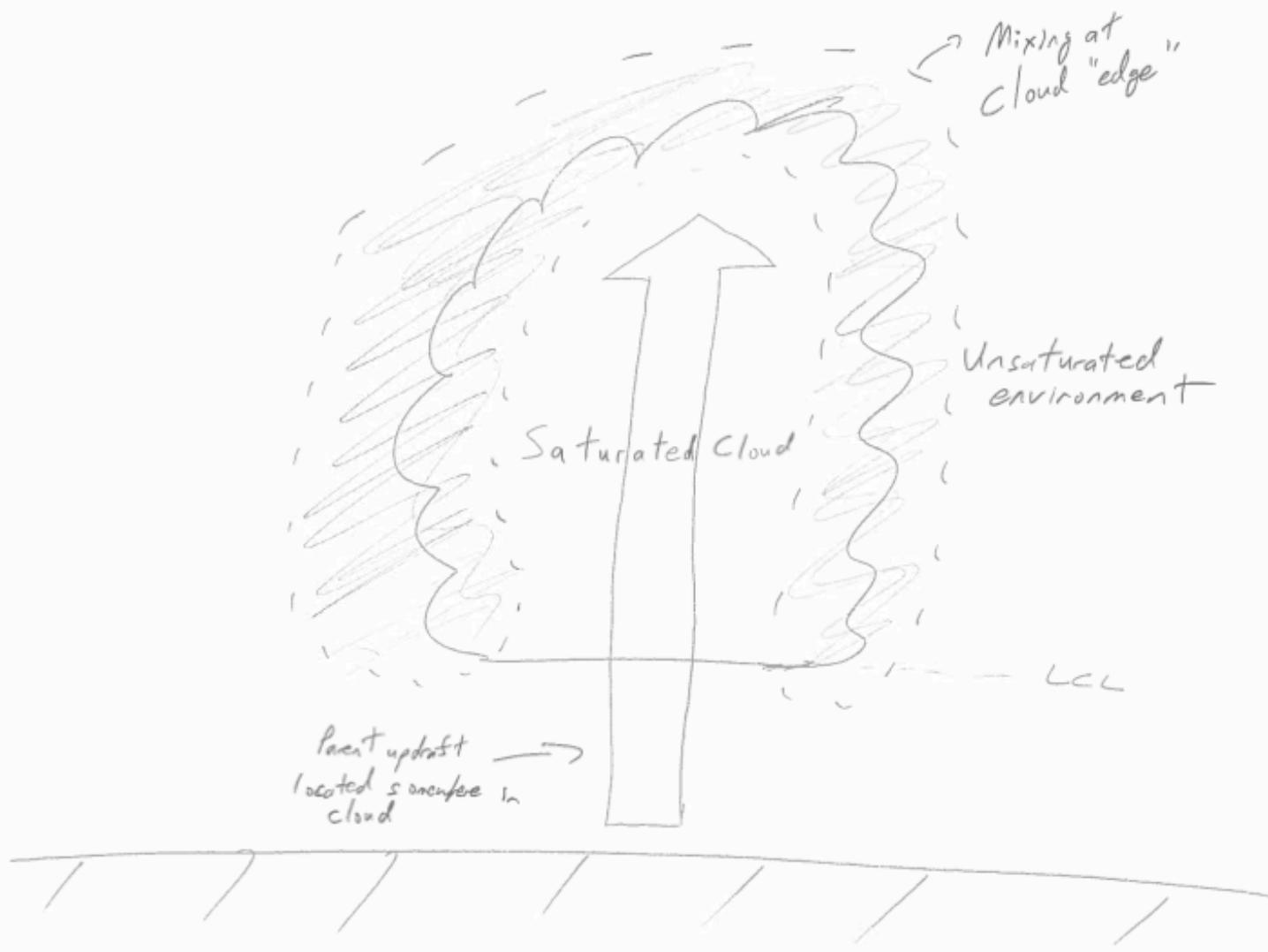


Satellite Data (Rushley et al. 2018)



$$CRH = \frac{\int_{P_{sfc}}^{P_{top}} q \, dP}{\int_{P_{sfc}}^{P_{top}} q_{sat}(T) \, dP}$$

But lots of scatter in rain rate at high CRH. Why?



Why does moisture impact deep convection? *Via entrainment.*

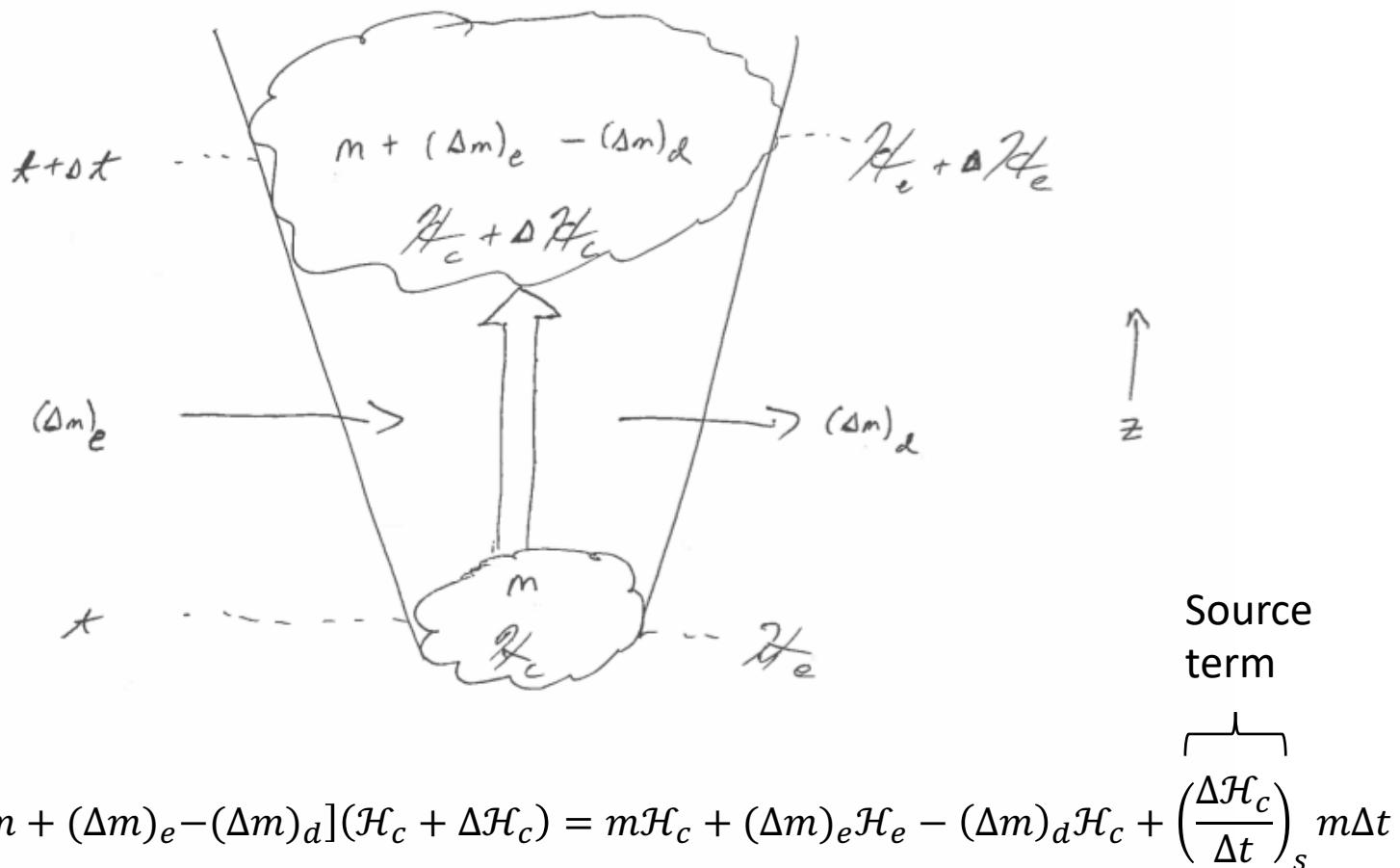
Definition of entrainment, from AMS Glossary:

In meteorology, the mixing of environmental air into a preexisting organized air current so that the environmental air becomes part of the current; the opposite of detrainment.

Entrainment of air into clouds, especially cumulus, is said to be inhomogeneous when the timescale for mixing of environmental air is very much greater than the timescale for drop evaporation. Under these conditions, which are often found when environmental air is first entrained into cumulus, regions of cloud and entrained air are intertwined, with evaporation occurring only on the edges of the interface between the cloudy and entrained environmental air.

Mathematically, consider the case of continuous, homogenous entrainment. This presumes that entrainment is an instant process that is experienced throughout a cloud in the same way.

See House (2014), Fig. 7.3
Stommel (1947)



Source
term

$$\left[m + (\Delta m)_e - (\Delta m)_d \right] (\mathcal{H}_c + \Delta \mathcal{H}_c) = m \mathcal{H}_c + (\Delta m)_e \mathcal{H}_e - (\Delta m)_d \mathcal{H}_c + \left(\frac{\Delta \mathcal{H}_c}{\Delta t} \right)_s m \Delta t$$

Take limit for
 $\Delta t \rightarrow 0$

$$\frac{D \mathcal{H}_c}{Dt} = \left(\frac{D \mathcal{H}_c}{Dt} \right)_s + \frac{1}{m} \left(\frac{Dm}{Dt} \right)_e (\mathcal{H}_e - \mathcal{H}_c)$$

What happens if we let
 \mathcal{H} represent moist
enthalpy ($h = c_p T + L_v q$)?

$$\frac{D h_c}{Dt} = M + \frac{1}{m} \left(\frac{Dm}{Dt} \right)_e (h_e - h_c)$$

Source/sink due to phase
changes (mainly
evaporation minus
condensation)

Difference in
temperature and/or
environmental vapor
and in-cloud vapor
concentrations.

Why is dry air detrimental to deep, moist convection?

$$\frac{Dh_c}{Dt} = M + \frac{1}{m} \left(\frac{Dm}{Dt} \right)_e (h_e - h_c)$$



If mass is entrained into cloud, then the 2nd term means that the moist enthalpy in the cloud decreases.

Difference in environmental and in-cloud temperature/humidity.

Consider equation for buoyancy (B) (here prime means deviation from hydrostatically balanced state):

$$B = -g \frac{\rho'}{\rho_0}$$

And the 3-D momentum equation neglecting Coriolis and friction:

$$\frac{\partial \mathbf{v}}{\partial t} = -\frac{1}{\rho_0} \nabla p' + B \mathbf{k} - \mathbf{v} \cdot \nabla \mathbf{v}$$

Changing $q_{v,c}$ alters ρ' . Water vapor (H_2O) is lighter than dry air (N_2 and O_2). **An decrease in water vapor increases ρ' and inhibits upward acceleration.**

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Other Thermodynamic Factors Impacting Moist Convection

Main Topics:

- Effects on convection by static stability
- Global distribution of lightning

What factors influence moist tropical convection? *Many, and the non-linear interactions between them are not fully understood!*

Leading factors impacting convection (**by altering buoyancy of convective updrafts**):

- 1) Moisture availability (A tautology to say that moist convection needs moisture, really. Of course, one needs moisture to support moist convection!)
- 2) Static stability
- 3) Surface fluxes
- 4) Wind shear (may impact buoyancy and affects structure of convection)
- 5) Low-level convergence (provides forcing)

However, these are impacted by other factors that are part of the large-scale dynamics that are themselves influenced by the convection:

- Advection
- Near-surface wind
- Sea surface temperature
- Adiabatically driven vertical motion
- Vertical flux of moisture (i.e. moist convection itself)
- Low-level convergence

Static Stability appears as S_p in the thermodynamic equation:

$$S_p = \frac{\Gamma_d - \Gamma}{\rho g} = -\frac{1}{c_p} \frac{\partial s}{\partial p}$$

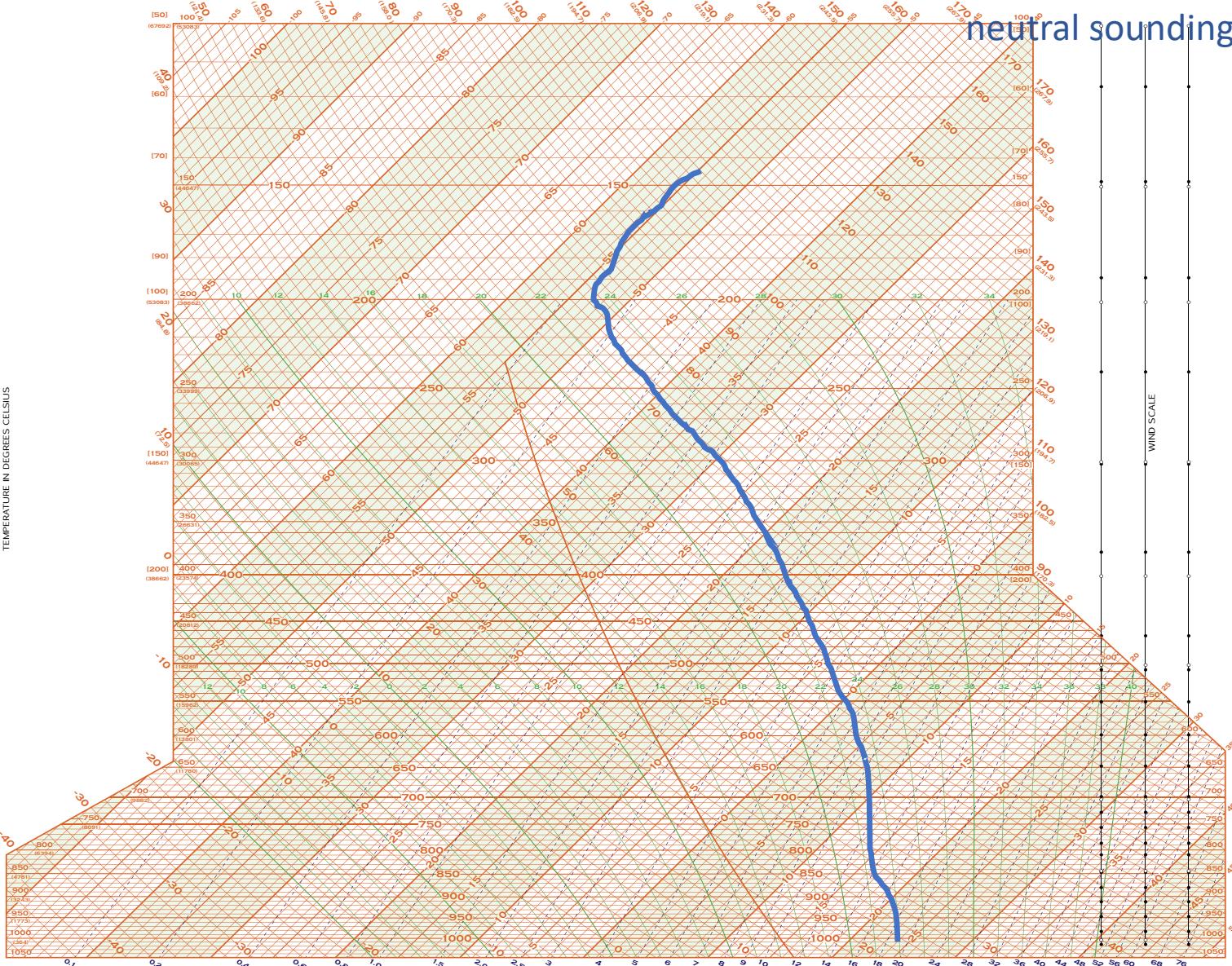
$$\frac{T}{\theta} \frac{\partial \theta}{\partial z} = \Gamma_d - \Gamma$$

$$s = c_p T + \phi \quad \longleftarrow \quad \text{Dry static energy}$$

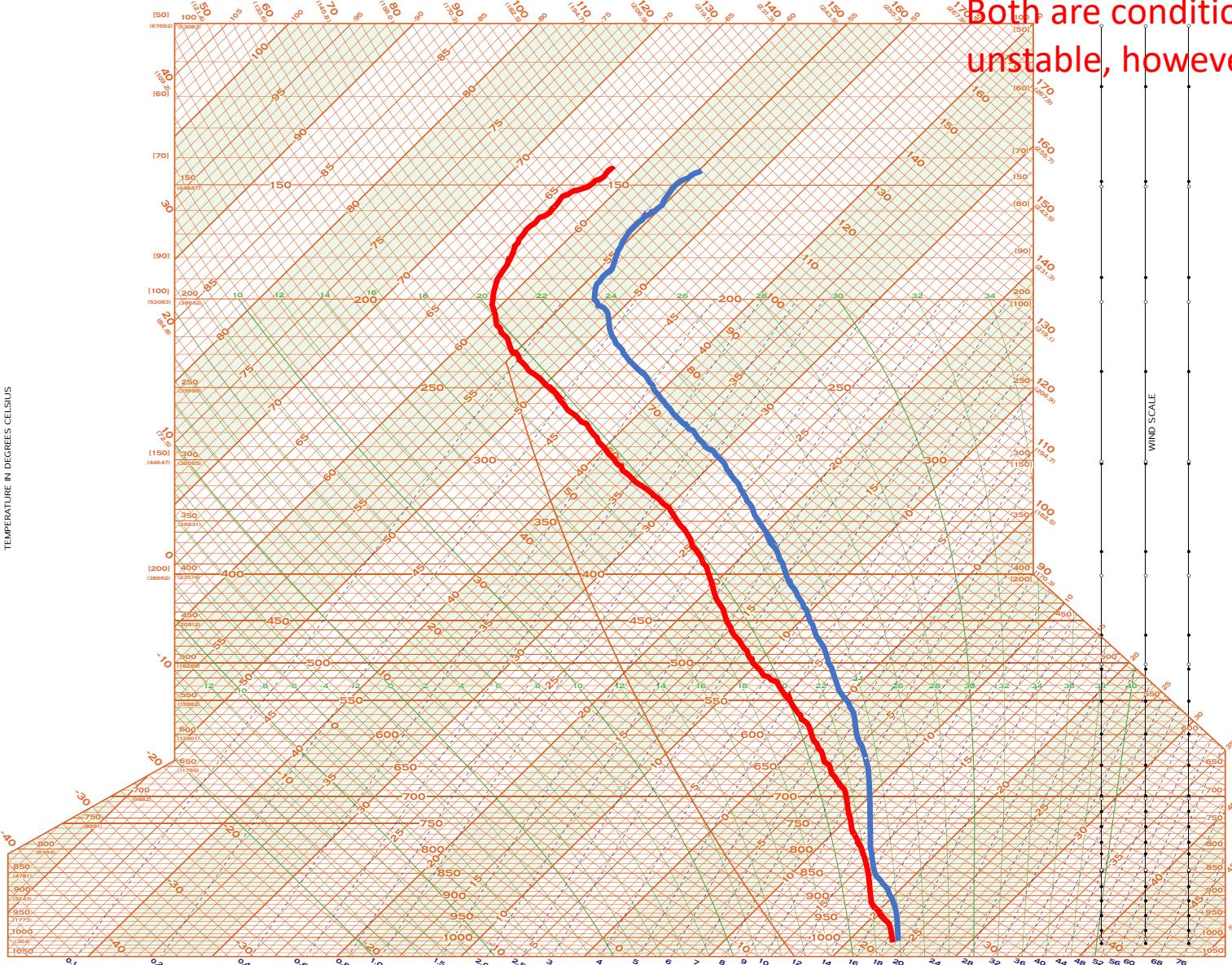
If the environmental lapse rate is large (i.e. atmosphere cools quickly with height), then air rising from the PBL is likely to be warmer (more buoyant) than the environment. A low lapse rate means the difference in environmental temperature and parcel temperature will be less (lower buoyancy; smaller vertical acceleration).

This is a conditionally unstable, but nearly moist neutral sounding.

SKEW T ADIABATIC DIAGRAM



SKEW T ADIABATIC DIAGRAM

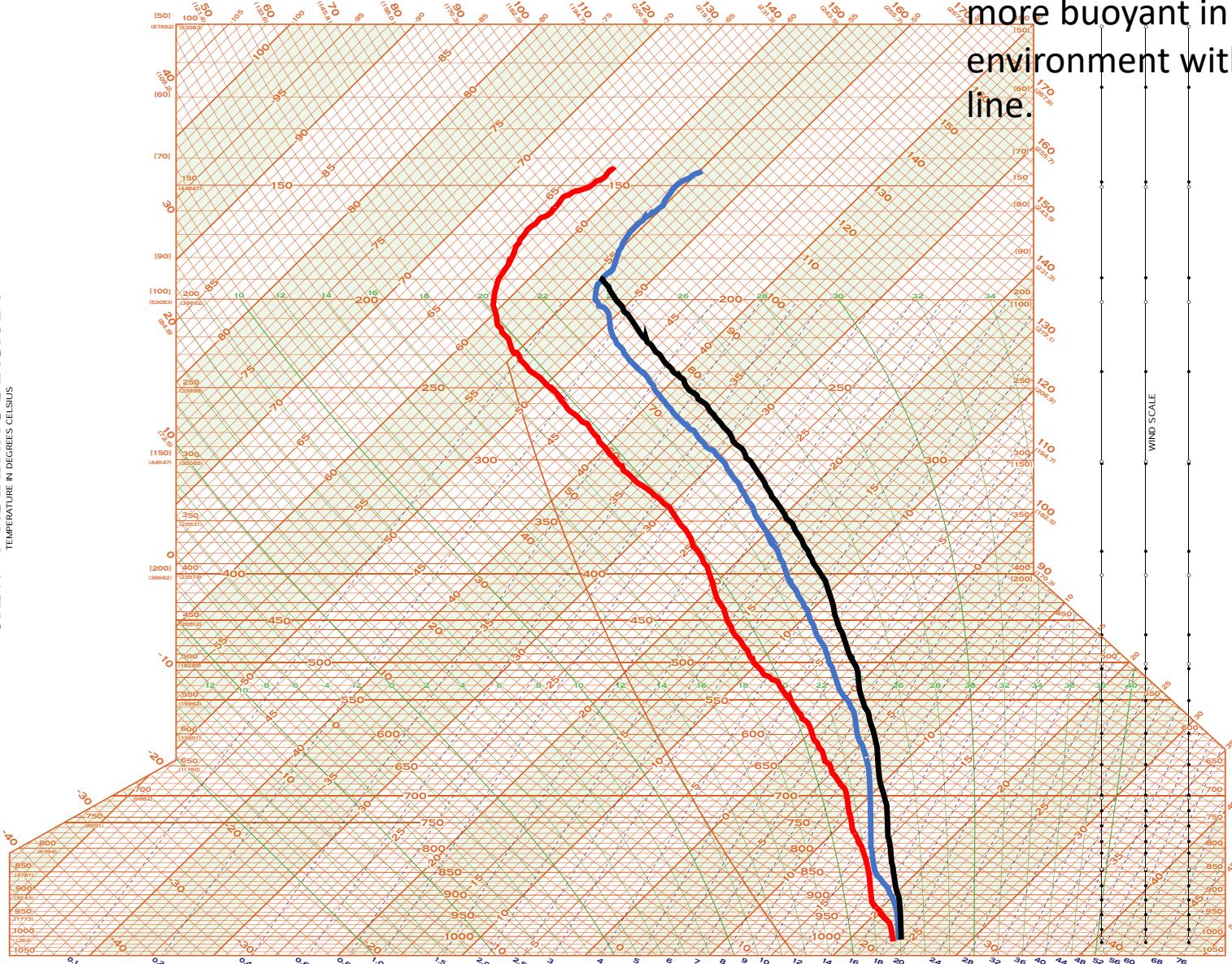


Red line is less stable than the blue line.

Both are conditionally unstable, however.

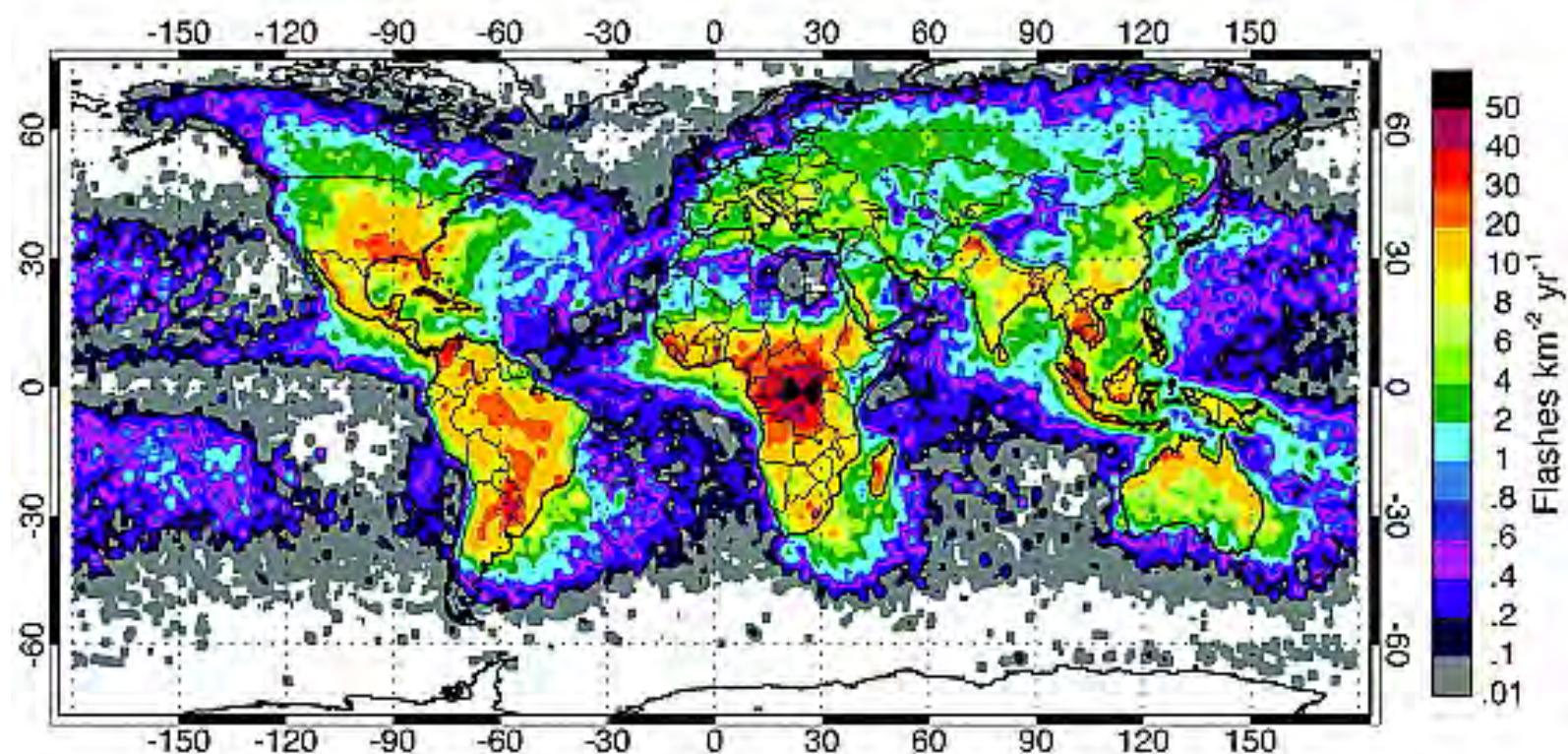
SKEW T ADIABATIC DIAGRAM

TEMPERATURE IN DEGREES CELSIUS



A parcel following the black line (a moist adiabat) will be more buoyant in an environment with the red line.

Global distribution of lightning: Far more common over land (Figure from Christian et al. 2003)



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Other Thermodynamic Factors Impacting Moist Convection

Main Topics:

- Effects on convection by surface fluxes
- Buoyancy flux
- Latent heat flux
- Sensible heat flux
- Global distribution of precipitation

Surface Fluxes

Two types of surface fluxes: Latent and sensible heat fluxes

Latent heat flux: Heat flux between atmosphere and surface associated with phase change of water. For example, evaporation of ocean water into the atmosphere would be a positive flux of latent heat to the atmosphere.

Sensible heat flux: Conductive heat flux between surface and atmosphere. If the surface is warmer than the surface-layer of the atmosphere, then the sensible heat flux to the atmosphere will be positive.

$$L = \rho L_v C_q U (q_s - q)$$

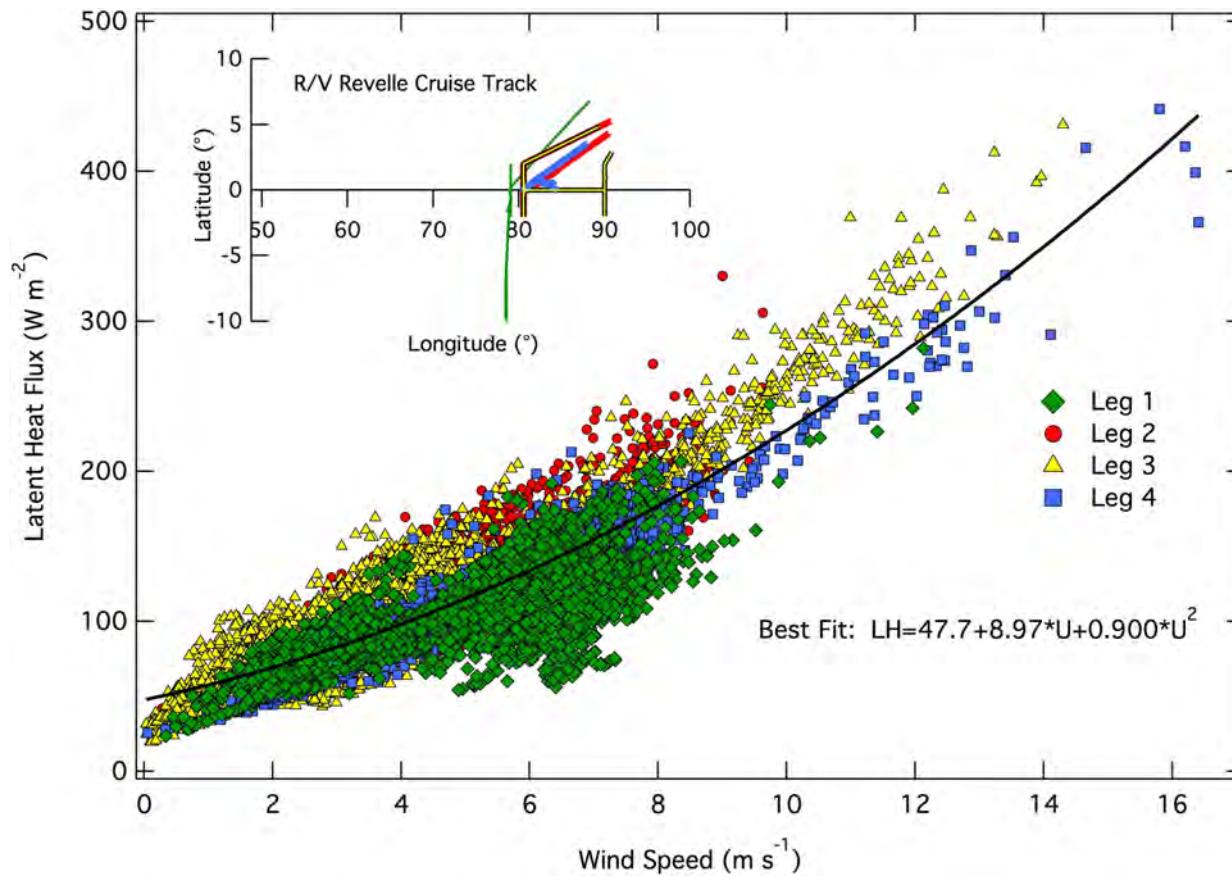
$$S = \rho c_p C_h U (\theta_s - \theta)$$

The C variables are exchange coefficients that are dependent on conditions at the air-sea interface.

Both fluxes are functions of wind, and the subscript s indicates the conditions of the ocean. q_s is the saturation specific humidity associated with air with the same temperature as the sea surface.

Surface Fluxes

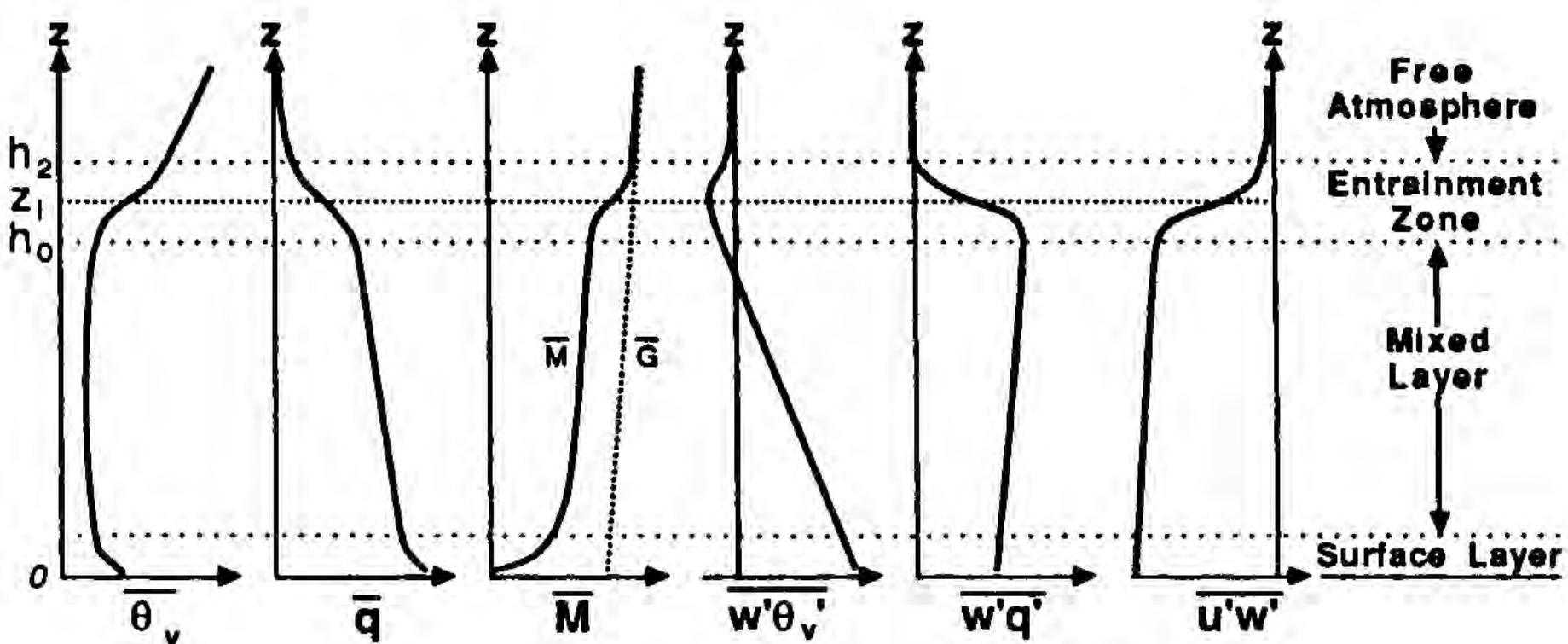
Qian et al.
(2016)



Surface latent heat fluxes vary non-linearly with near-surface wind speed. Wind will alter surface roughness, which impacts exchange coefficient. Therefore, impact of wind on surface fluxes is nonlinear!

Typical profiles of quantities in a convective boundary layer

$$\theta_v \approx \theta(1 + 0.61r), r = \text{mixing ratio}$$



mean virtual
potential
temperature

mean
specific
humidity

mean wind
speed
buoyancy
flux

specific
humidity
flux

momentum
flux

What is a buoyancy flux?

"The vertical kinematic flux of virtual potential temperature $\overline{w'\theta_v'}$, which when multiplied by the buoyancy parameter (g/T_v) yields a flux that is proportional to buoyancy. (AMS Glossary)

$$\text{Buoyancy Flux} = \frac{g}{T_v} \overline{w'\theta_v'}$$

Turbulent sensible heat flux:

$$LHF = \rho_0 L_v \overline{w'q_v'}$$

$$SHF = \rho_0 c_p \overline{w'\theta'}$$

Take the equation for virtual potential temperature (ignoring hydrometeors, which we presume are not near the surface):

$$\theta_v \approx \theta(1 + 0.61r), r = \text{mixing ratio}$$

We eventually arrive at

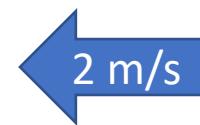
$$\text{Buoyancy Flux} = \frac{g}{\rho c_p T} \left(SHF + 0.61 \underbrace{\frac{c_p T}{L_v} LHF }_{\approx 0.08} \right)$$

For typical temperatures, the contributions of SHF and LHF to buoyancy flux are similar in magnitude, even if $LHF \gg SHF$.

Well-mixed PBL

$$T = 28^\circ\text{C}$$

$$q = 18 \text{ g/kg}$$



SST = 25°C

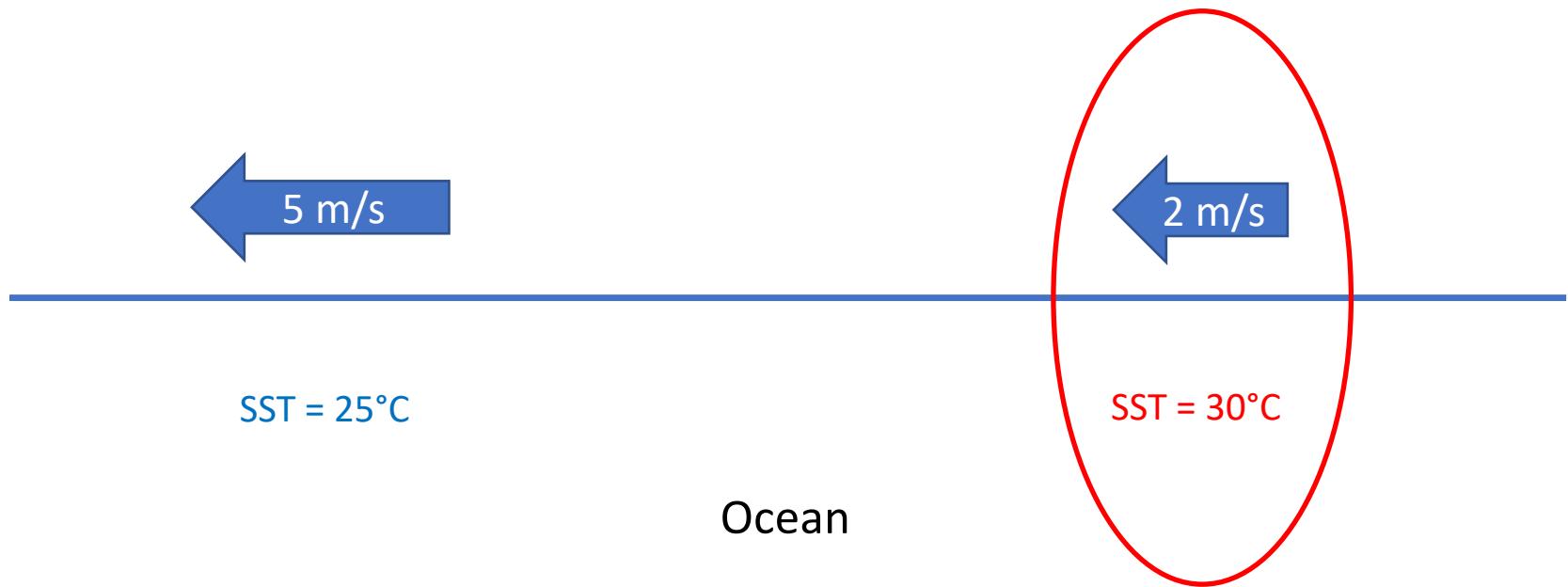
SST = 30°C

Ocean

Well-mixed PBL

$$T = 28^\circ\text{C}$$

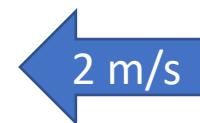
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Well-mixed PBL

$$T = 28^\circ\text{C}$$

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SST = 30°C

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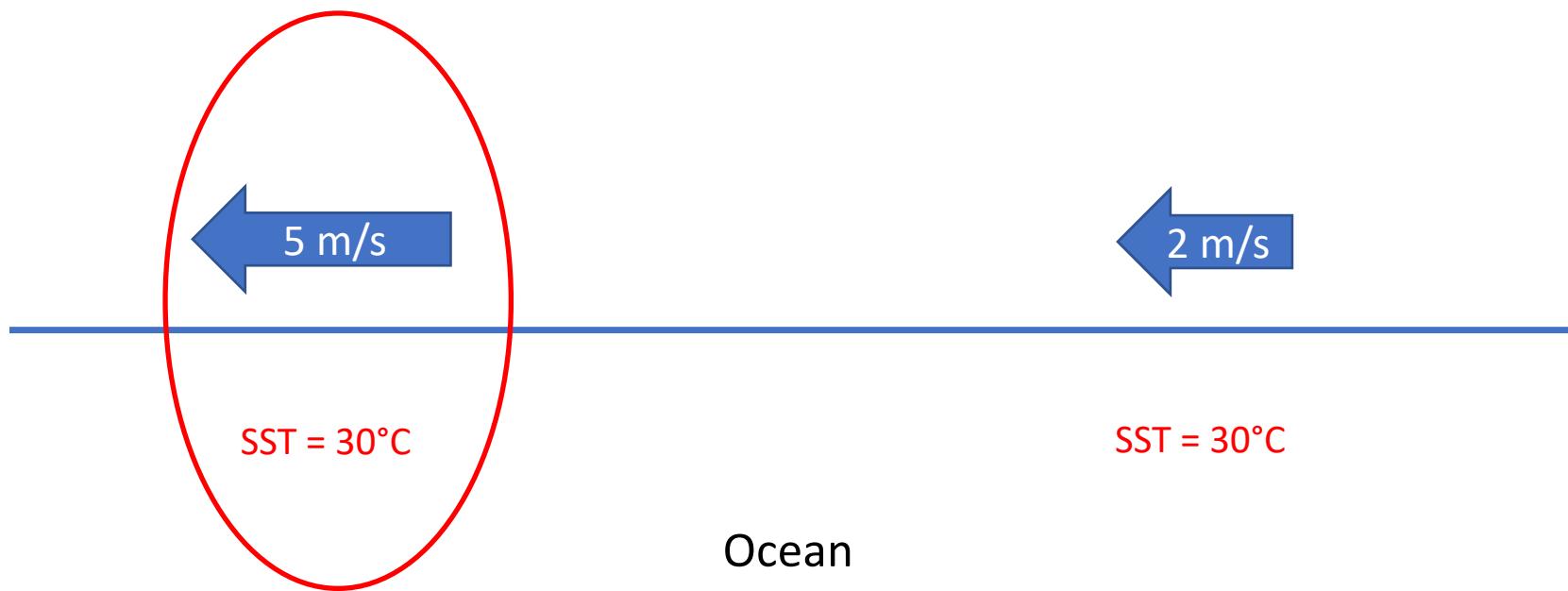
Ocean

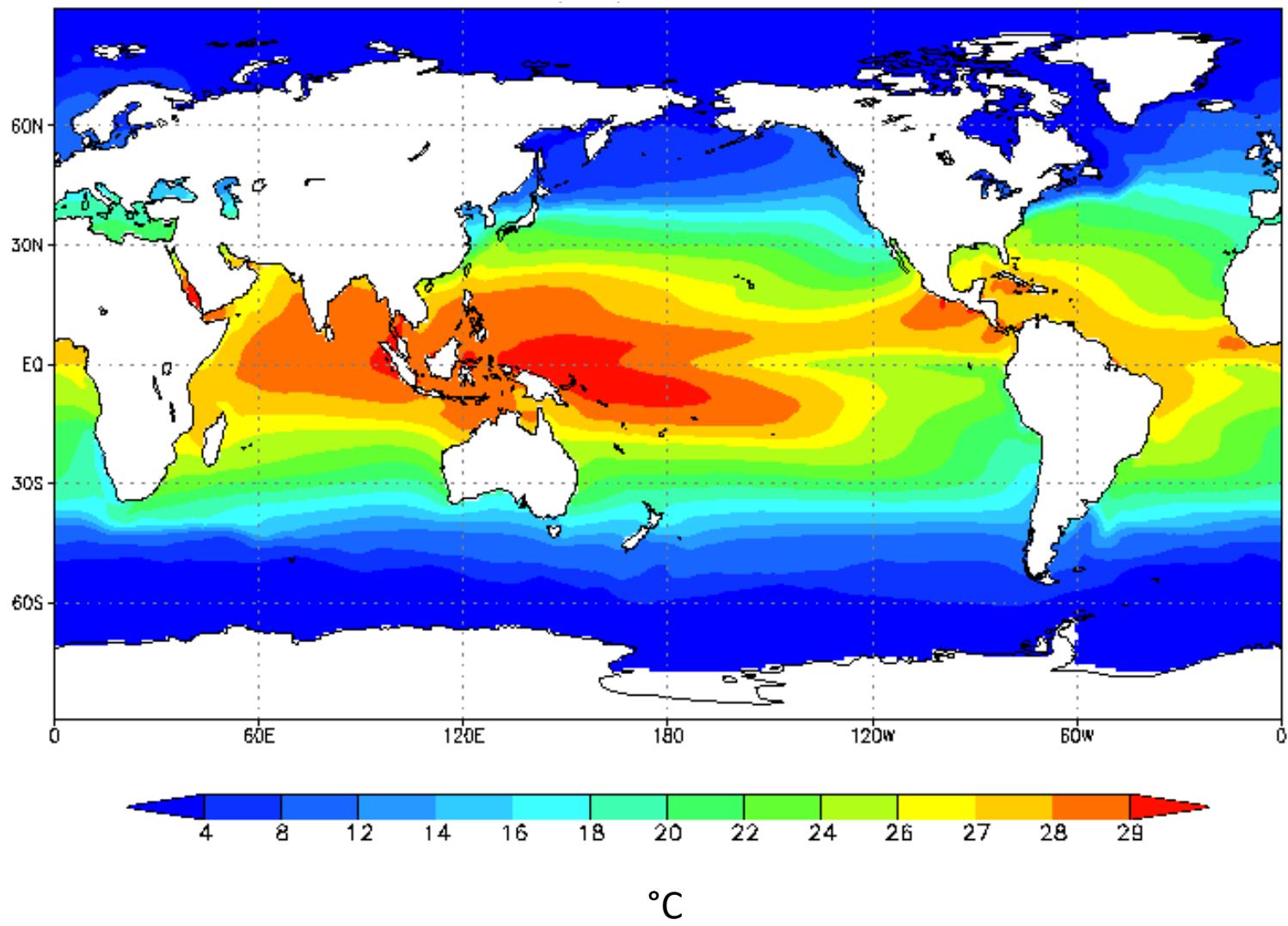
Generally, high surface wind and high sea surface temperature will lead to an increase in boundary layer moist static energy (unless the PBL is already very warm and moist).

Well-mixed PBL

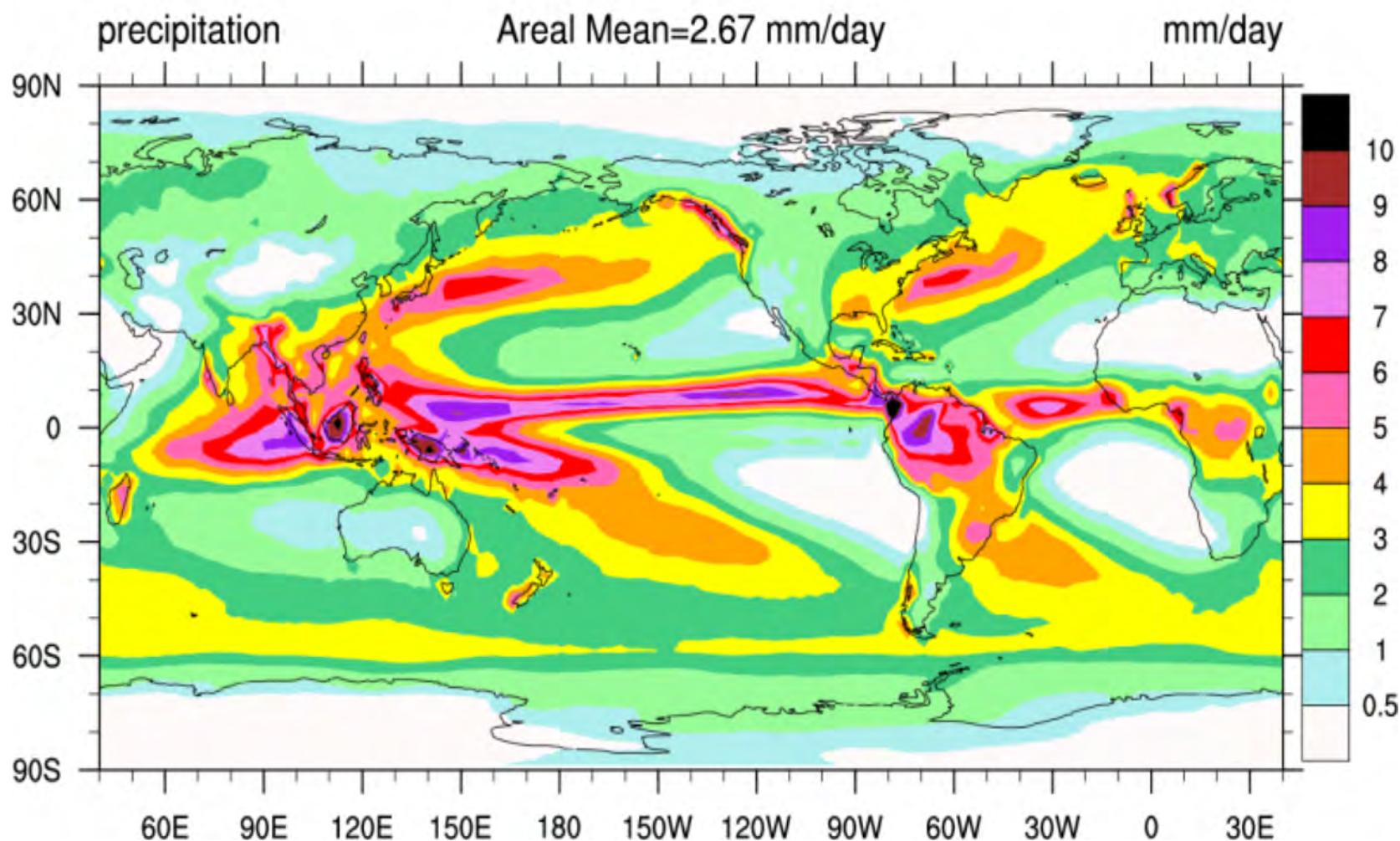
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TRMM GPCP: 1979-2010



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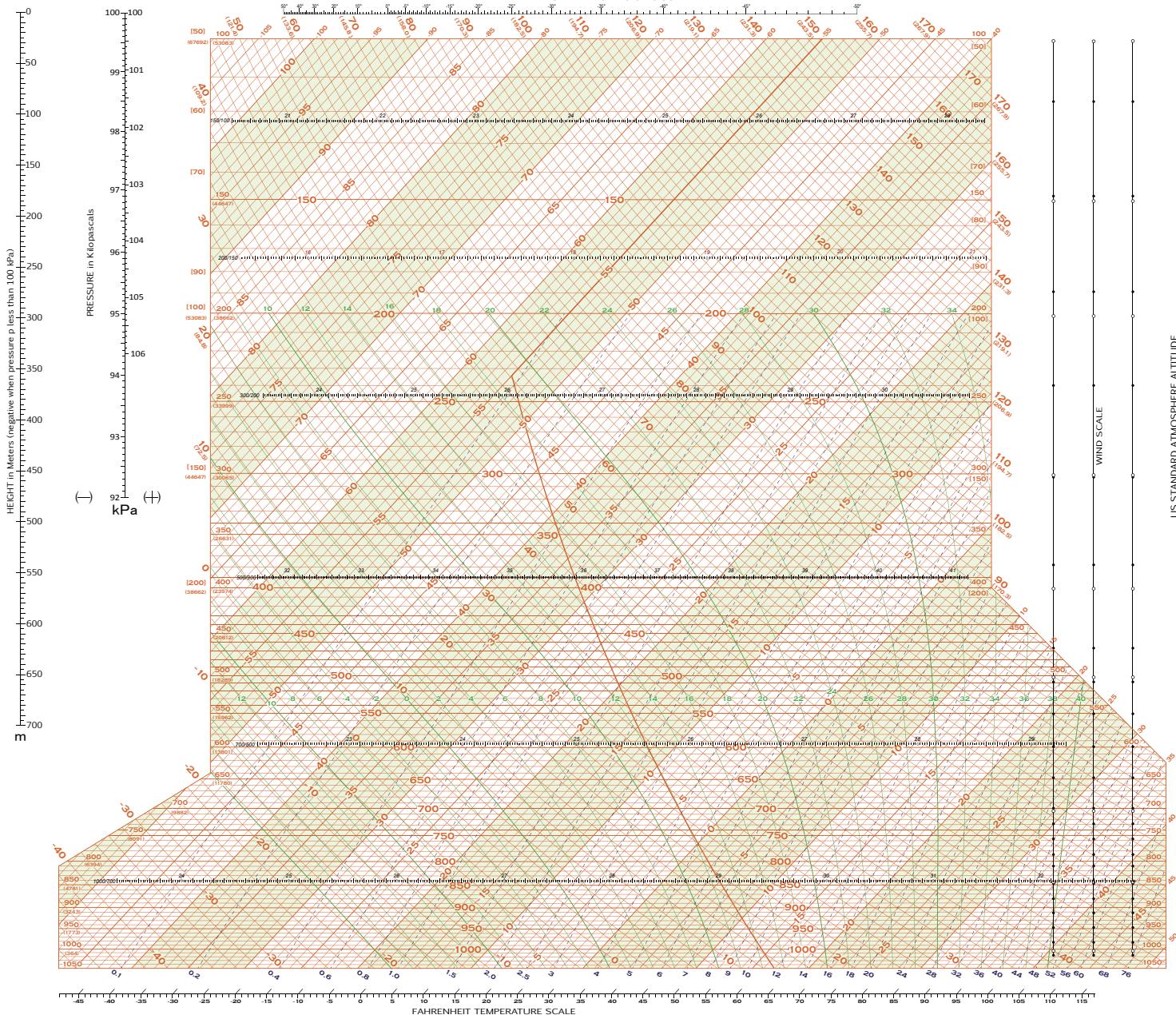
Thermodynamic Profiles and Important Quantities in the Tropical Atmosphere, Part I

Main Topics:

- Reading a Skew T-log P plot
- Potential temperature
- Equivalent potential temperature
- Diagnosing convective instability

SKEW T ADIABATIC DIAGRAM

TEMPERATURE IN DEGREES CELSIUS



EXPLANATION
 ISOBARS are straight horizontal brown lines. The heights in feet of the pressure surfaces in the U.S. Standard atmosphere are in parentheses () below the pressure values on the left.
 ISOADIABATS are the slightly curved blue lines that intersect the 1000 mb isobar at intervals of 2°C, and run diagonally upward from right to left. The temperatures in the labeled portion of the pressure range are labeled with two (2) digits. (See below).

SATURATED ADIABATS are the curved green lines that intersect the 1000 mb isobar at intervals of 2°C, and run diagonally upward and to the right from left to right. The temperatures in the labeled portion of the pressure range are labeled with two (2) digits. (See below).

SATURATION MIXING RATIO (in gm. per kg.) is represented by dashed green lines. Their values appear at the bottom of diagram.

THICKNESS (in hundreds of geopotential meters) of the layers between the levels 1000 mb and 850 mb is indicated by the numbers (in parentheses) and a graduation along the middle of each layer. The thicknesses are obtained from the virtual temperature curve by the equal-area method, using any straight line to divide the area.

HEIGHT in geopotential meters above mean sea level, or station level, of the 1000 mb surface is obtained from the nomogram in the upper left-hand corner by reading the pressure at the station level on the temperature scale (°C) through the point p (mean sea level or station pressure) on the pressure scale, and reading the height on the height scale.

U.S. STANDARD ATMOSPHERE SOUNDSINGS is indicated by a thick brown line.

The saturated adiabats and isopleths of saturation mixing ratio are computed by use of vapor pressure over a plane water surface at all temperatures.

Extensions of curves to 50 mb, has been accomplished by overlap with pressure surfaces. The values of the extension are given in parentheses [] at 50 mb. Dry adiabats for the overlap are labeled in parentheses. ().

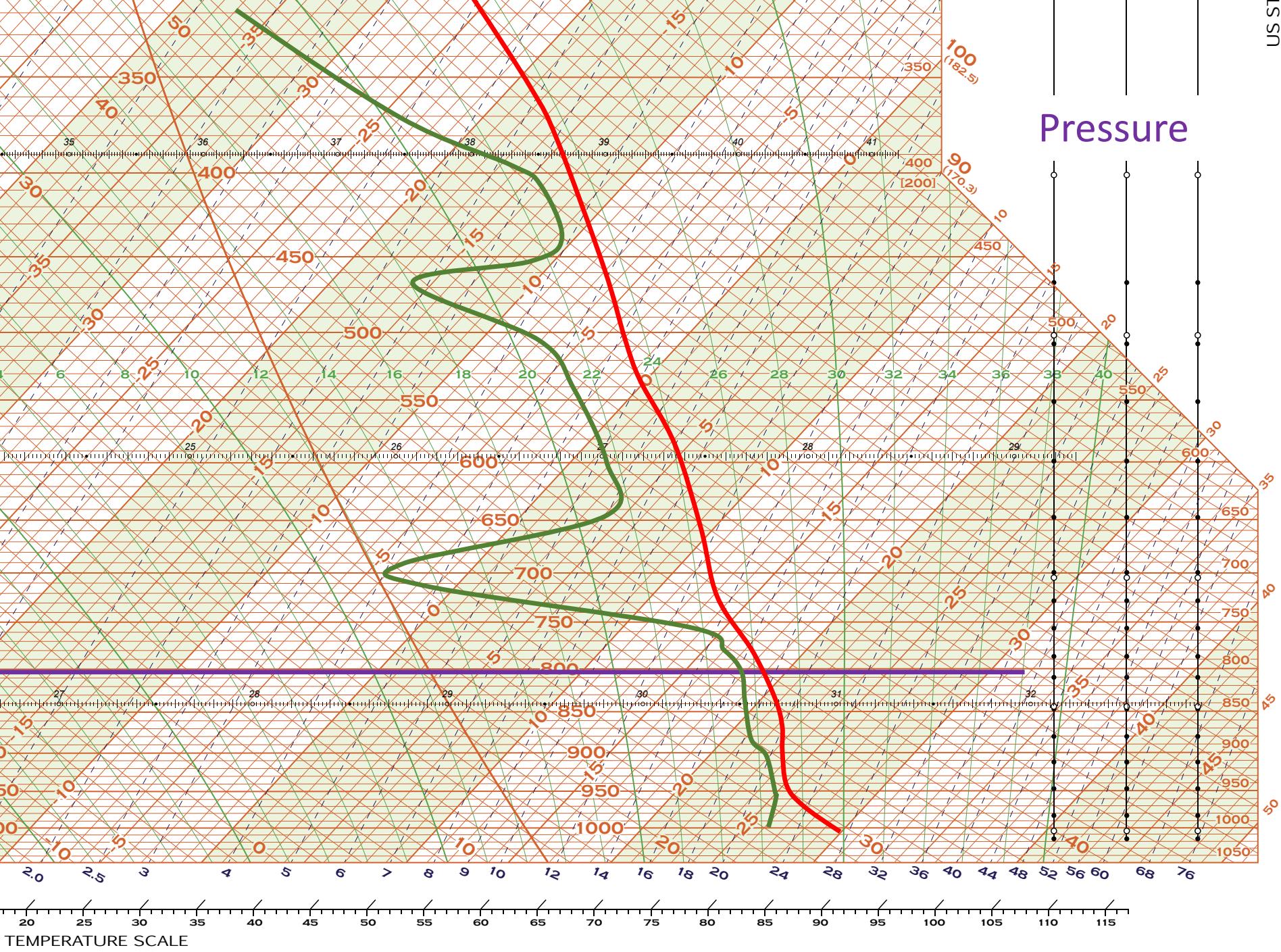
APPROXIMATE VIRTUAL TEMPERATURE may be obtained from the formula $T_v = T + \frac{p_s}{p} \cdot 10^3$, where T_v is virtual temperature in °C, T is air temperature in °C, and p_s is vapor pressure in mb. For the purpose of thickness computation, use the mean temperature of the layer for T and use the mean mixing ratio of the layer for p_s .

Blue and black double lines indicate the levels for which wind data is recorded and plotted. The open circles indicate the mandatory pressure level at which wind data is also entered.

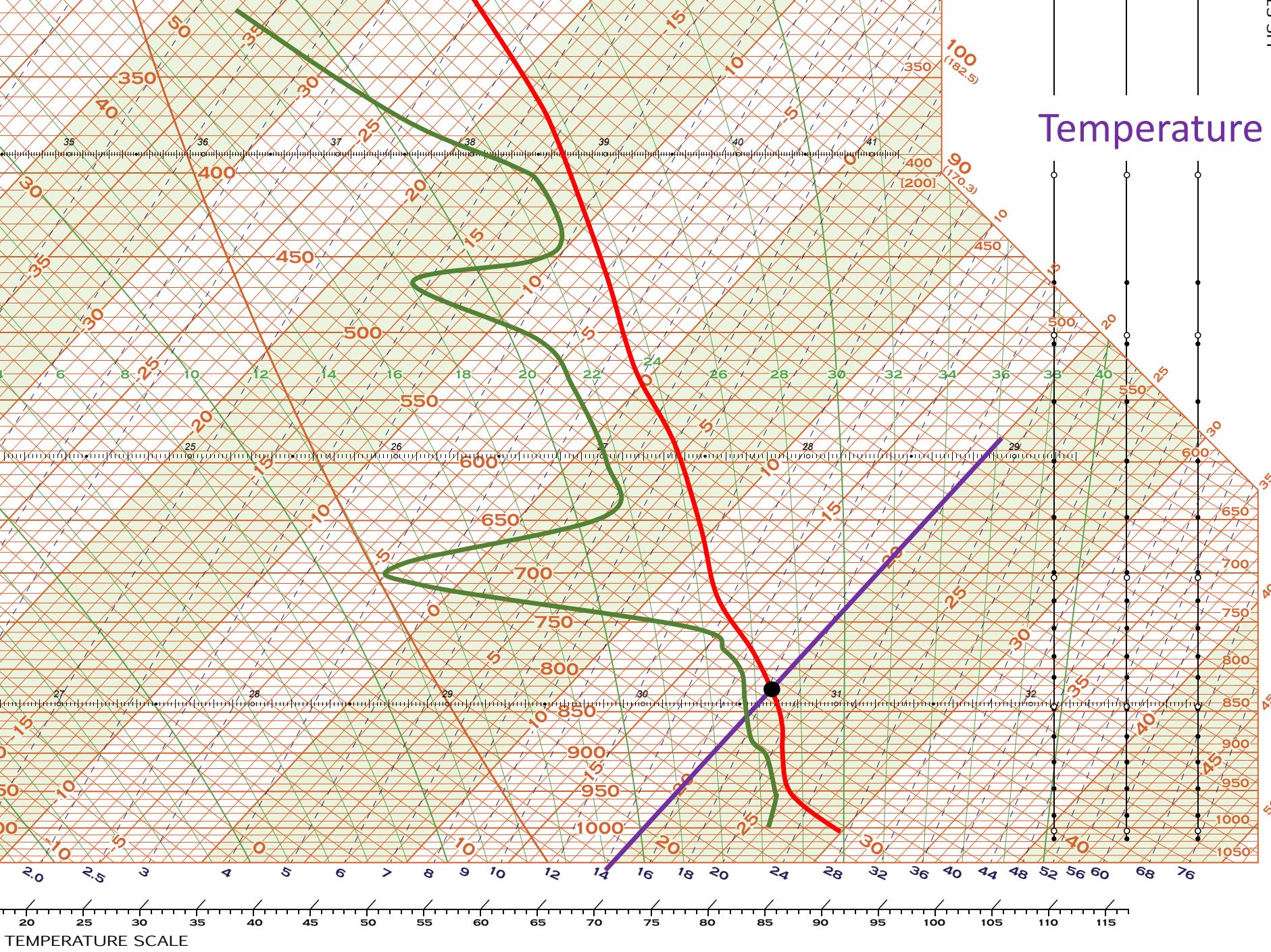
| SKEW T ANALYSIS | |
|---------------------------------------|------|
| TIME | TIME |
| ARMED ANALYSIS | |
| TYPE | M |
| TYPE | M |
| TYPE | M |
| LEVEL | |
| FRONTAL | |
| INVERSIONS | |
| SUBSIDENCE | |
| DEPRESSIONS | |
| LCL | |
| CUL | |
| W.D. | |
| WIND | |
| WIND | |
| LEVEL OF SHEAR | |
| STABILITY | |
| INDEX | |
| TO | TO |
| TO | TO |
| TO | TO |
| TYPE | |
| AMOUNT | |
| MASS | |
| GPS | |
| ICING | |
| TYPE | |
| INTENSITY | |
| BOUNDRNRS | |
| PERSISTENCE | |
| HEIGHT | |
| DIMES | |
| HIGH DENSITY | |
| VAL. HGT | |
| DATA | |
| MIN | |
| CUMULUS CLOUD FORMATION AT TEMP | TIME |
| DESCRIPTION OF LOW LEVEL INVERSION AT | TIME |
| REMARKS | |

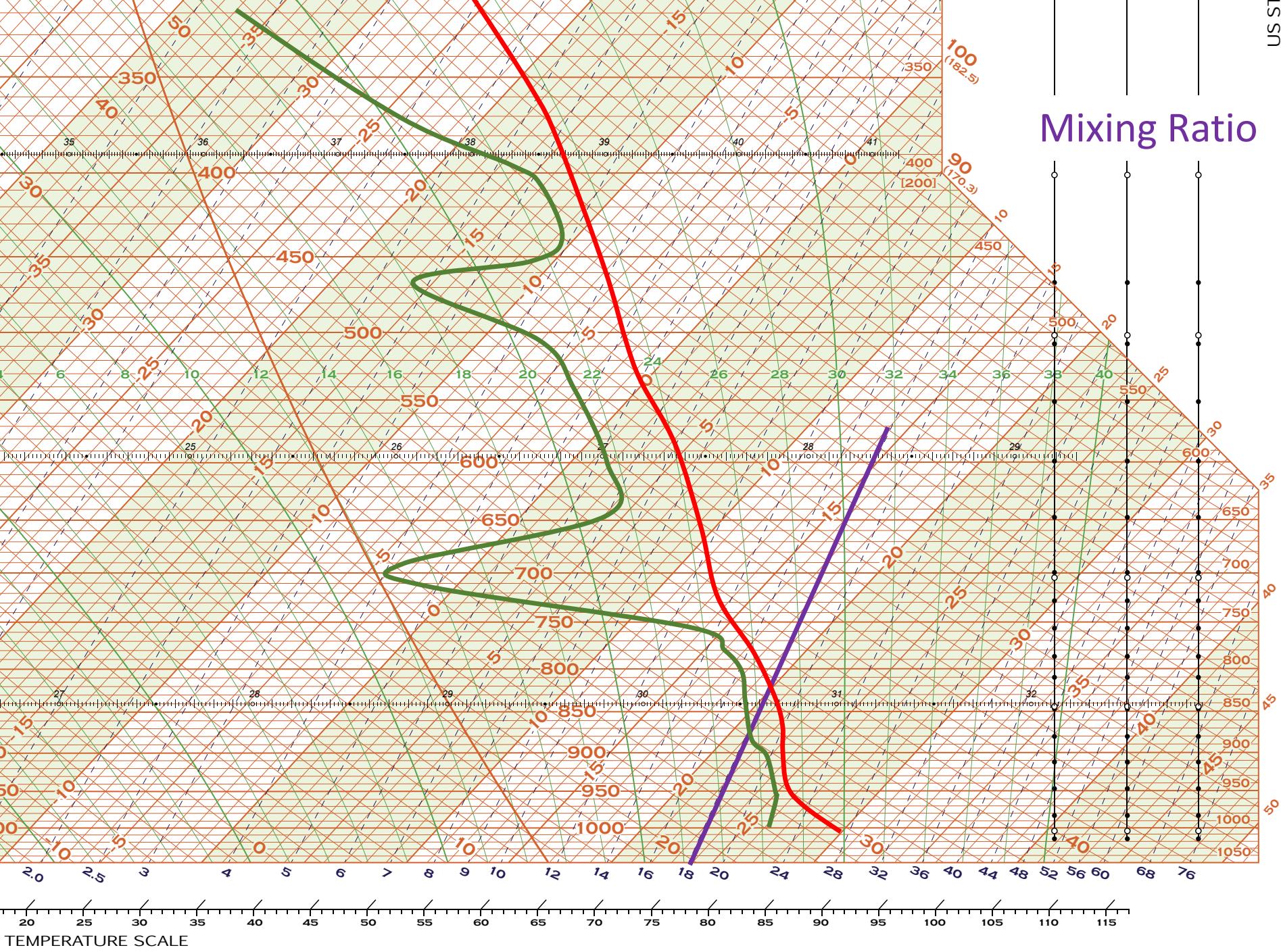
| STATION ID | STATION ID | STATION ID |
|------------|------------|------------|
| DATE | DATE | DATE |
| TIME (GMT) | TIME (GMT) | TIME (GMT) |

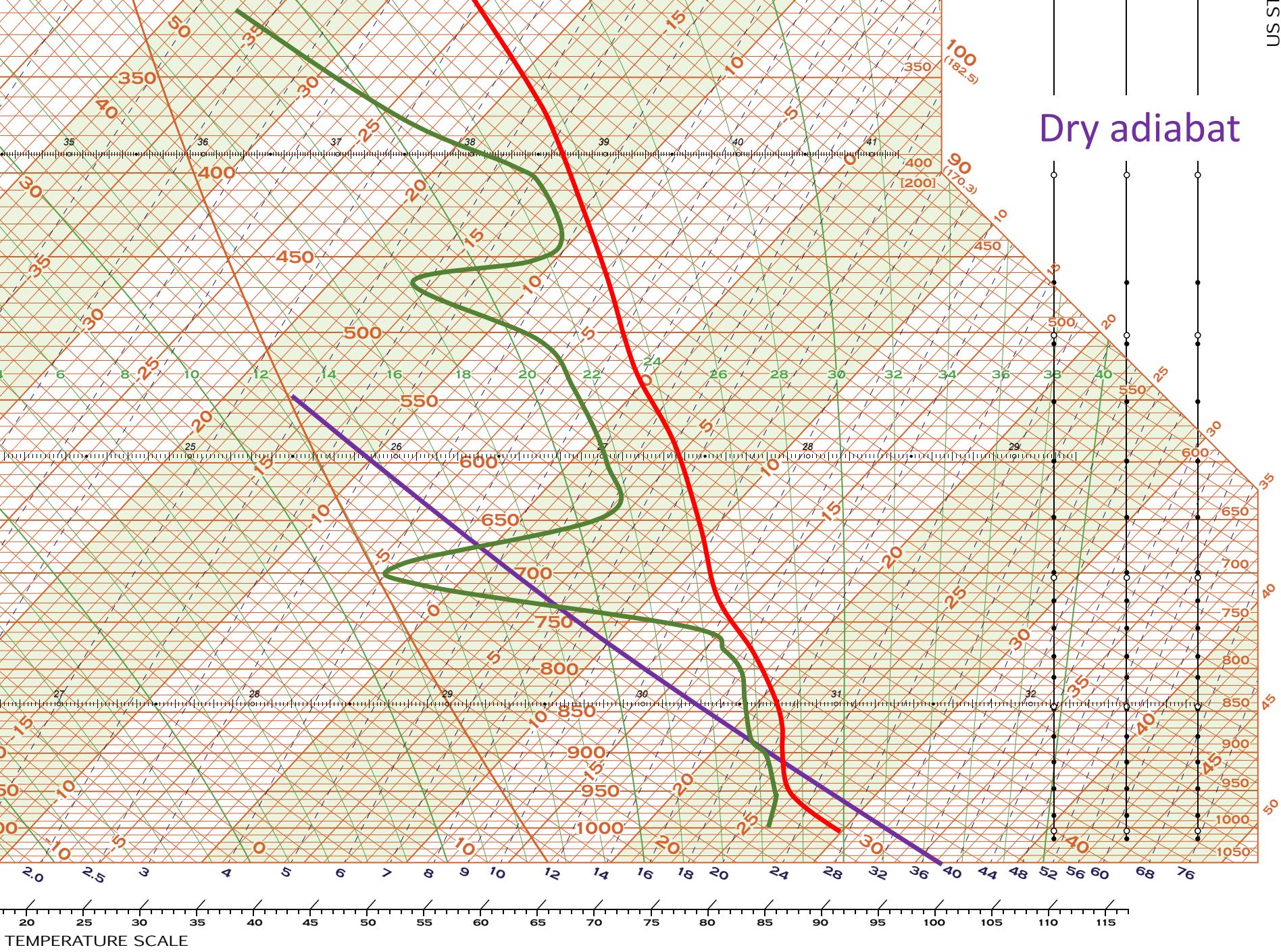
CHART D-7
NOAA - National Weather Service 1977 Jan



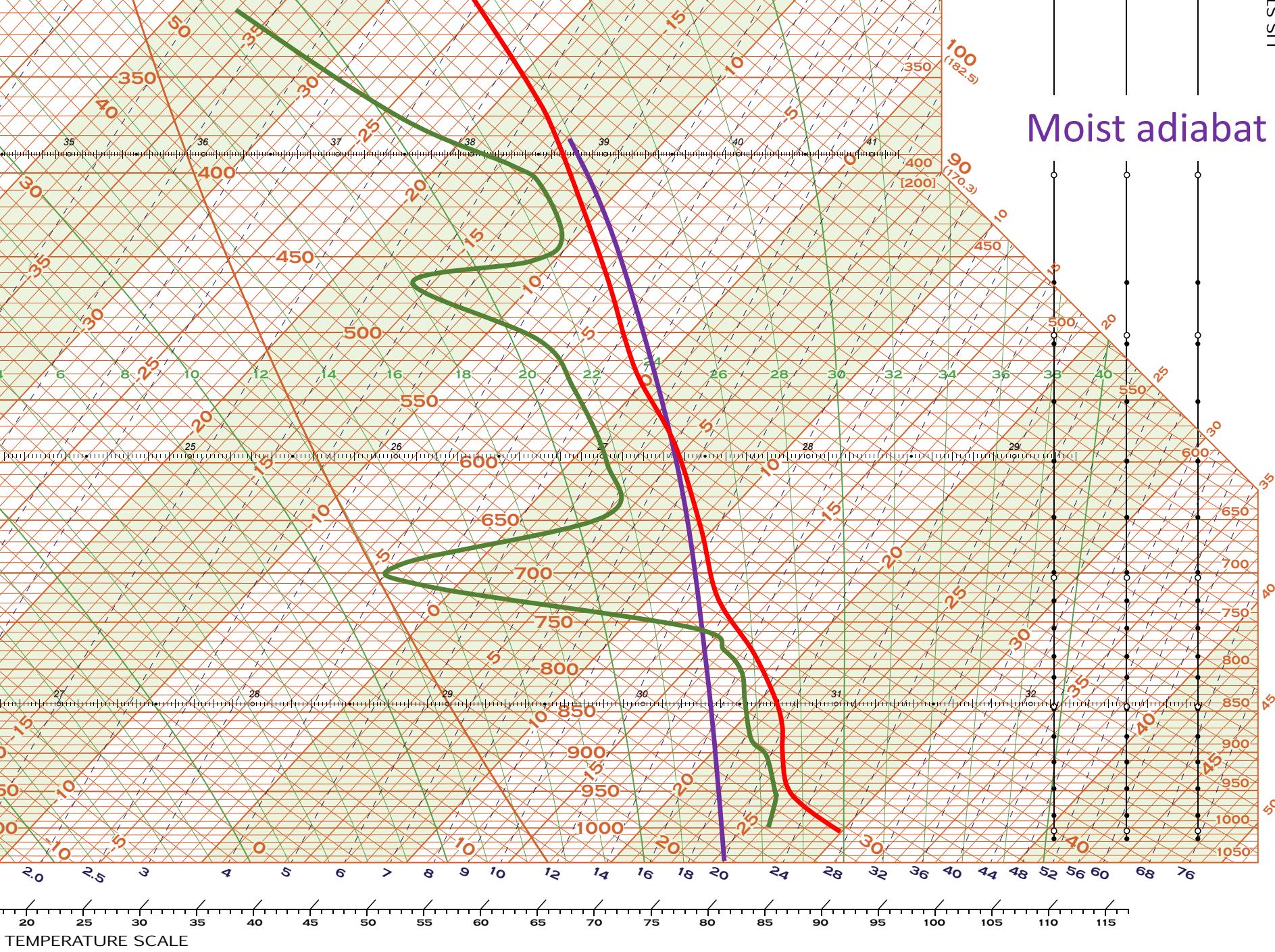
Temperature







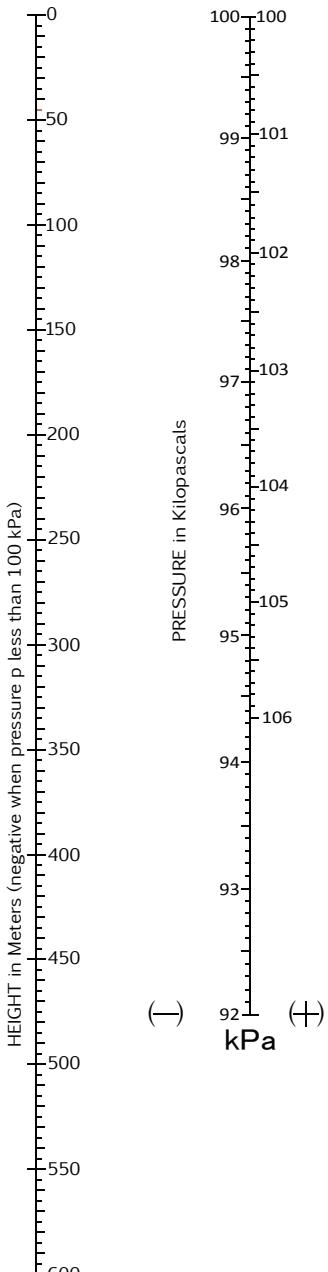
Moist adiabat



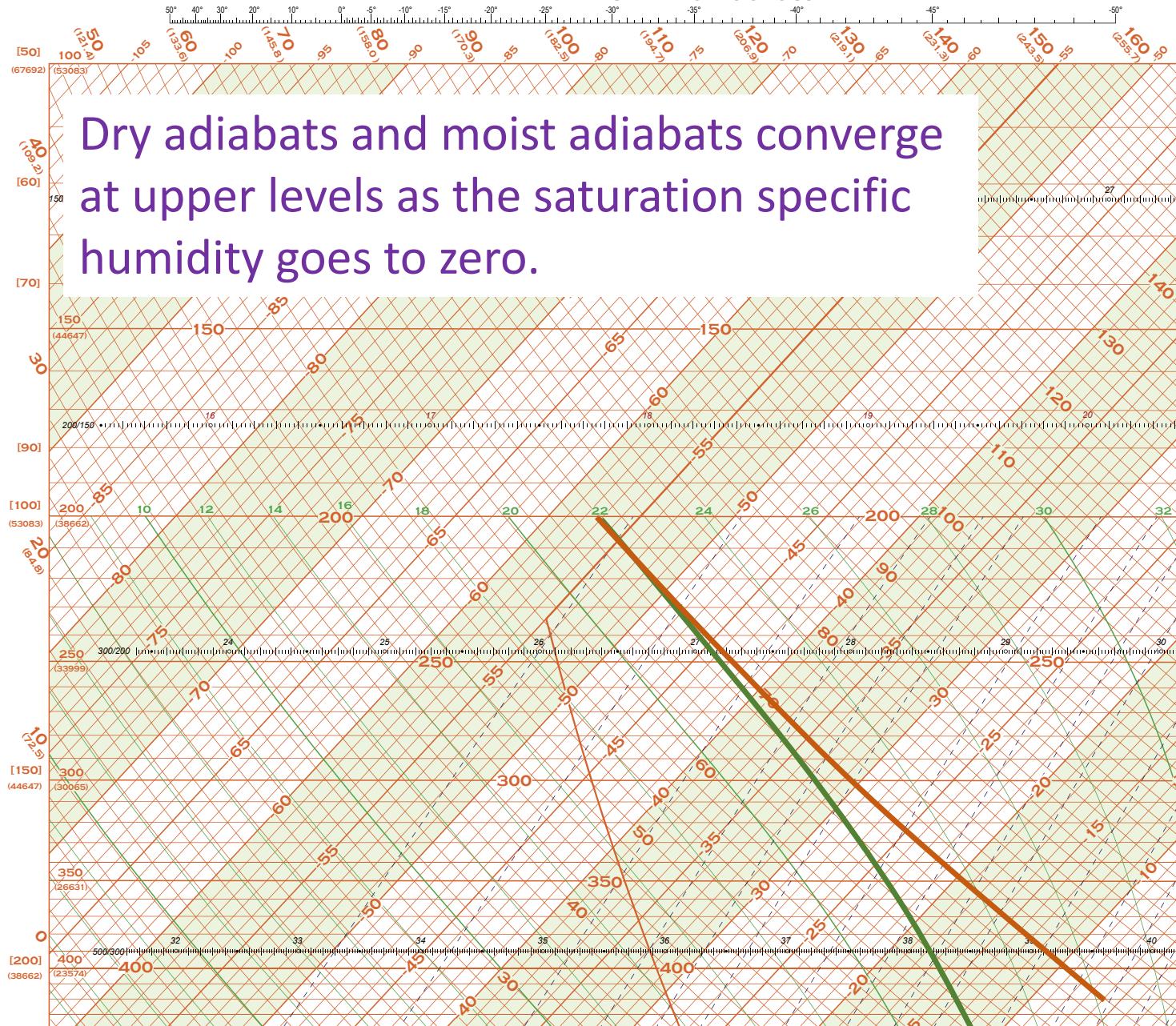


SKEW T ADIABATIC DIAGRAM

TEMPERATURE IN DEGREES CELSIUS



Dry adiabats and moist adiabats converge at upper levels as the saturation specific humidity goes to zero.



Some quantities with which to be familiar:

Potential temperature (θ): Temperature of air dry adiabatically lifted or descended to 1000 hPa.

Equivalent potential temperature (θ_e): Temperature of air if lifted adiabatically until all water vapor is condensed, then descended dry adiabatically to 1000 hPa.

Saturation equivalent potential temperature (θ_{es}): Same as θ_e , but assuming air is initially saturated. $\theta_{es} \geq \theta_e$.

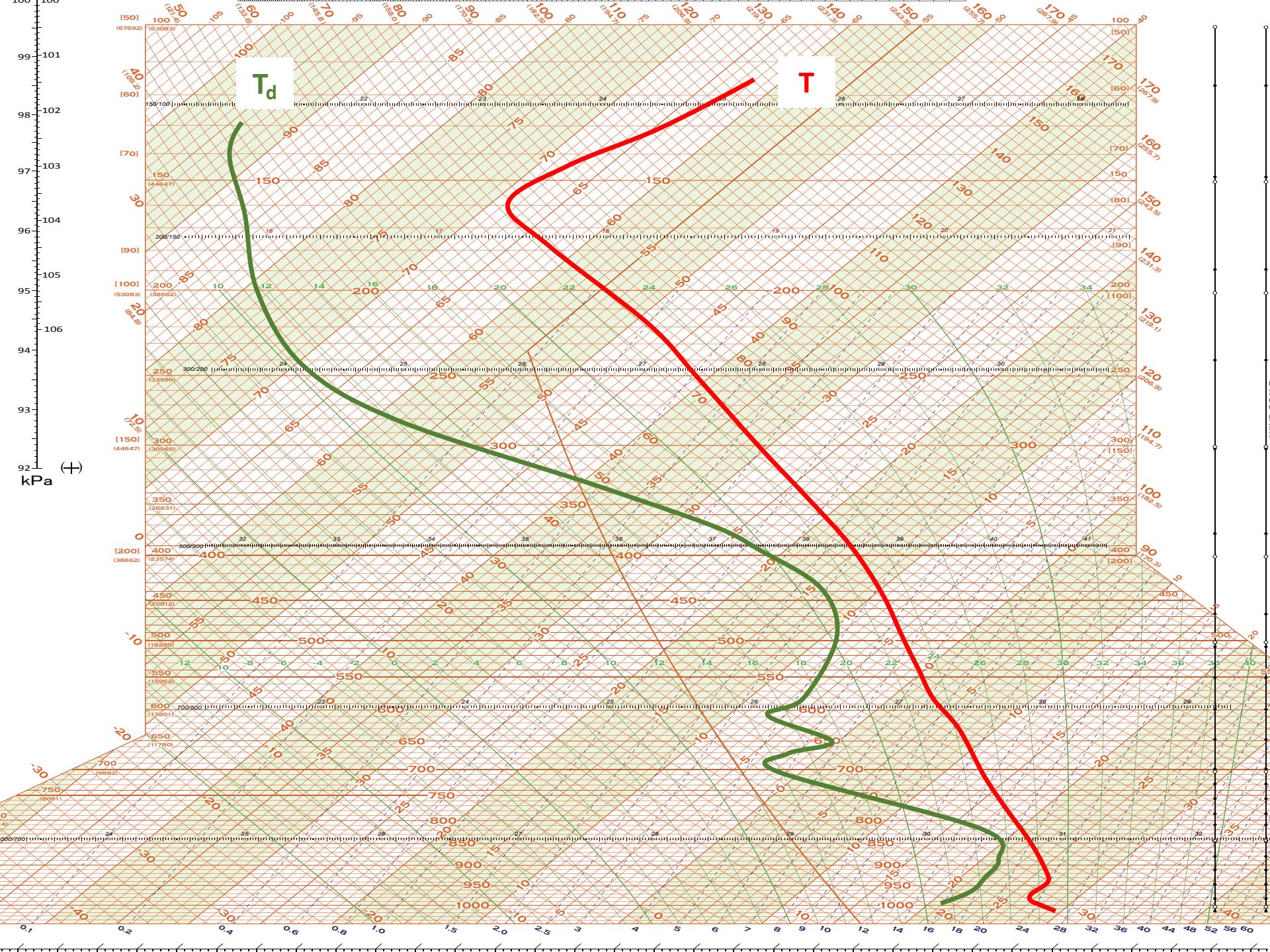
Virtual potential temperature (θ_v): Potential temperature of dry air that would have the same density as moist air. In the Tropics, may be O(1K) larger than θ .

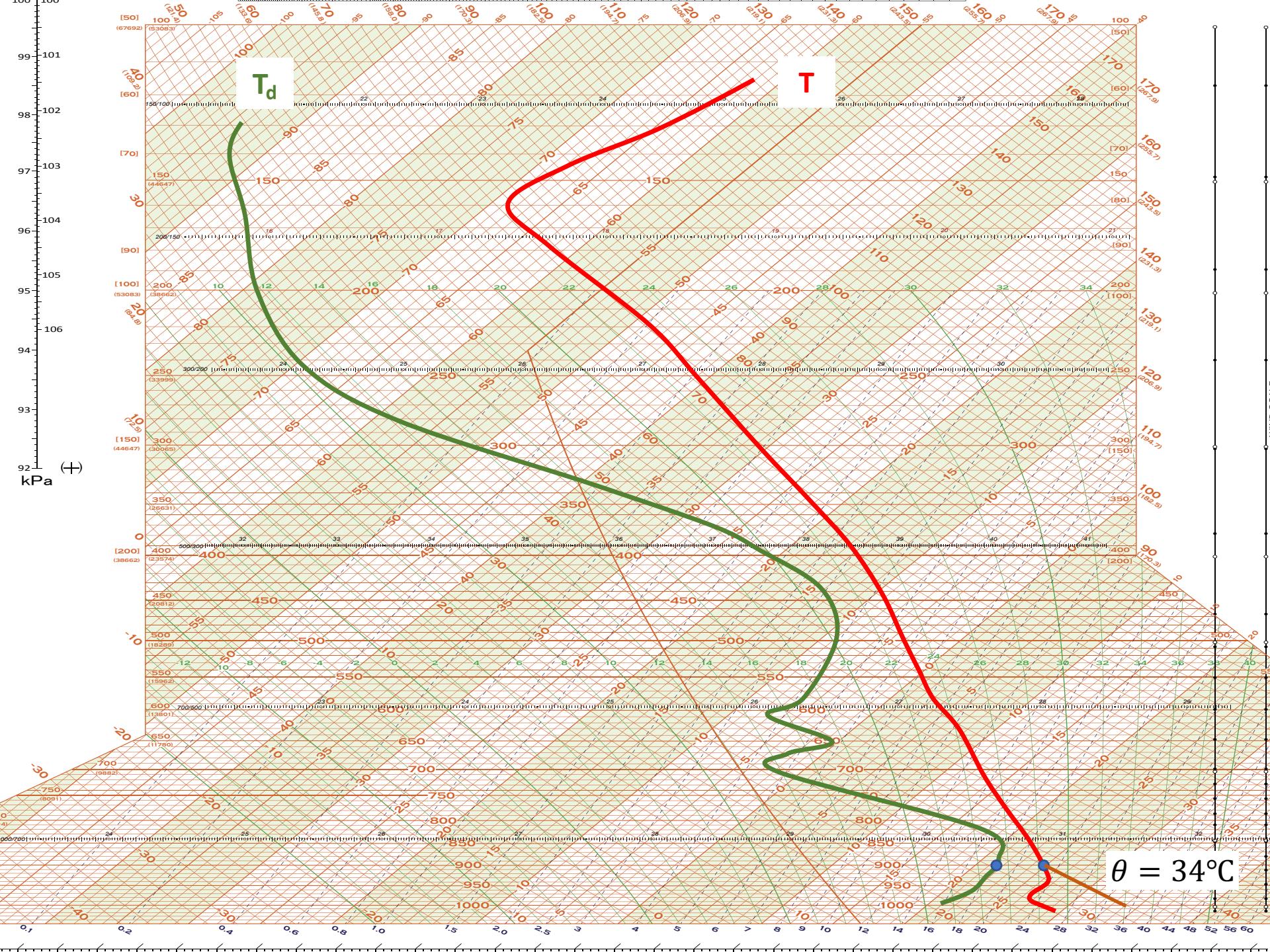
Environmental lapse rate: Observed change of temperature or a potential temperature variable with height.

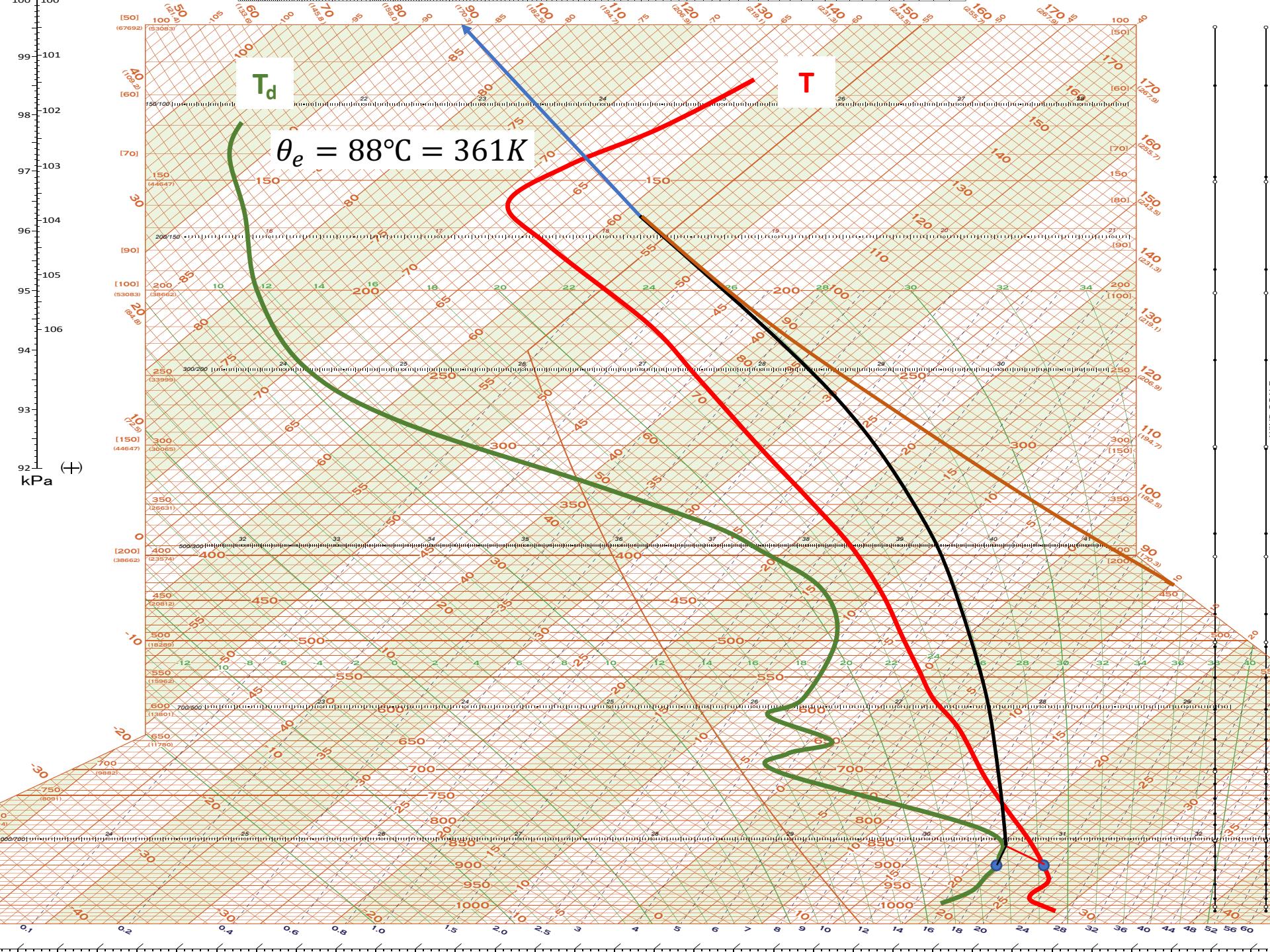
Dry adiabatic lapse rate: Adiabatic lapse rate that applies to sub-saturated air: $\Gamma_d = \frac{g}{c_p} \approx 9.81\text{K/km}$

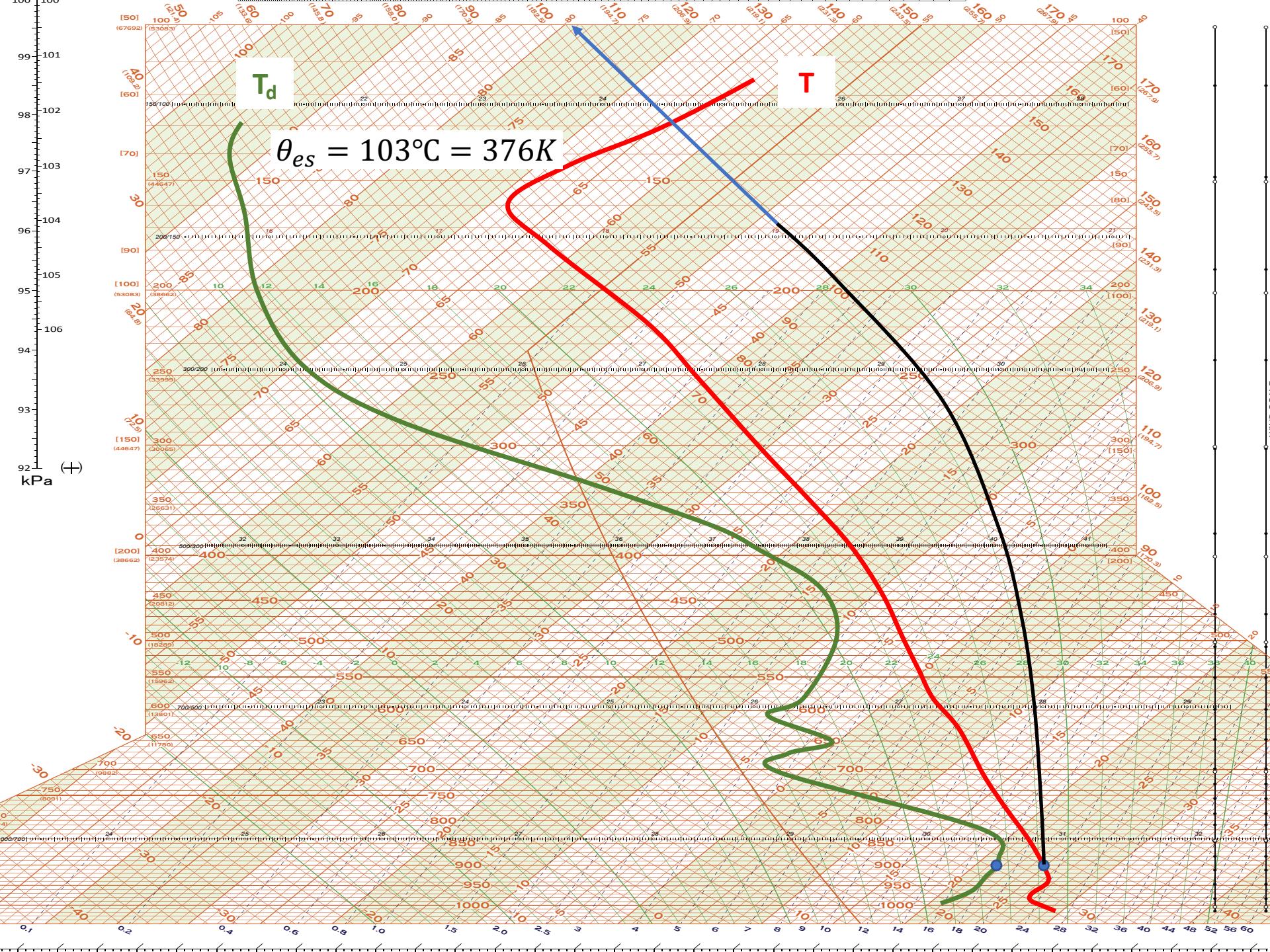
Moist adiabatic lapse rate: Adiabatic lapse rate that applies to saturated air:

$$\Gamma_m = g \frac{1 + \frac{L_v r}{RT}}{c_p + \frac{\epsilon L_v^2 r}{RT^2}}$$









Some quantities with which to be familiar:

Potential temperature (θ): Temperature of air dry adiabatically lifted or descended to 1000 hPa.

Equivalent potential temperature (θ_e): Temperature of air if lifted (dry) adiabatically until all water vapor is condensed, then descended (moist) adiabatically to 1000 hPa.

Saturated equivalent potential temperature (θ_{es}): Same as θ_e , but assuming air is initially saturated. $\theta_{es} \geq \theta_e$.

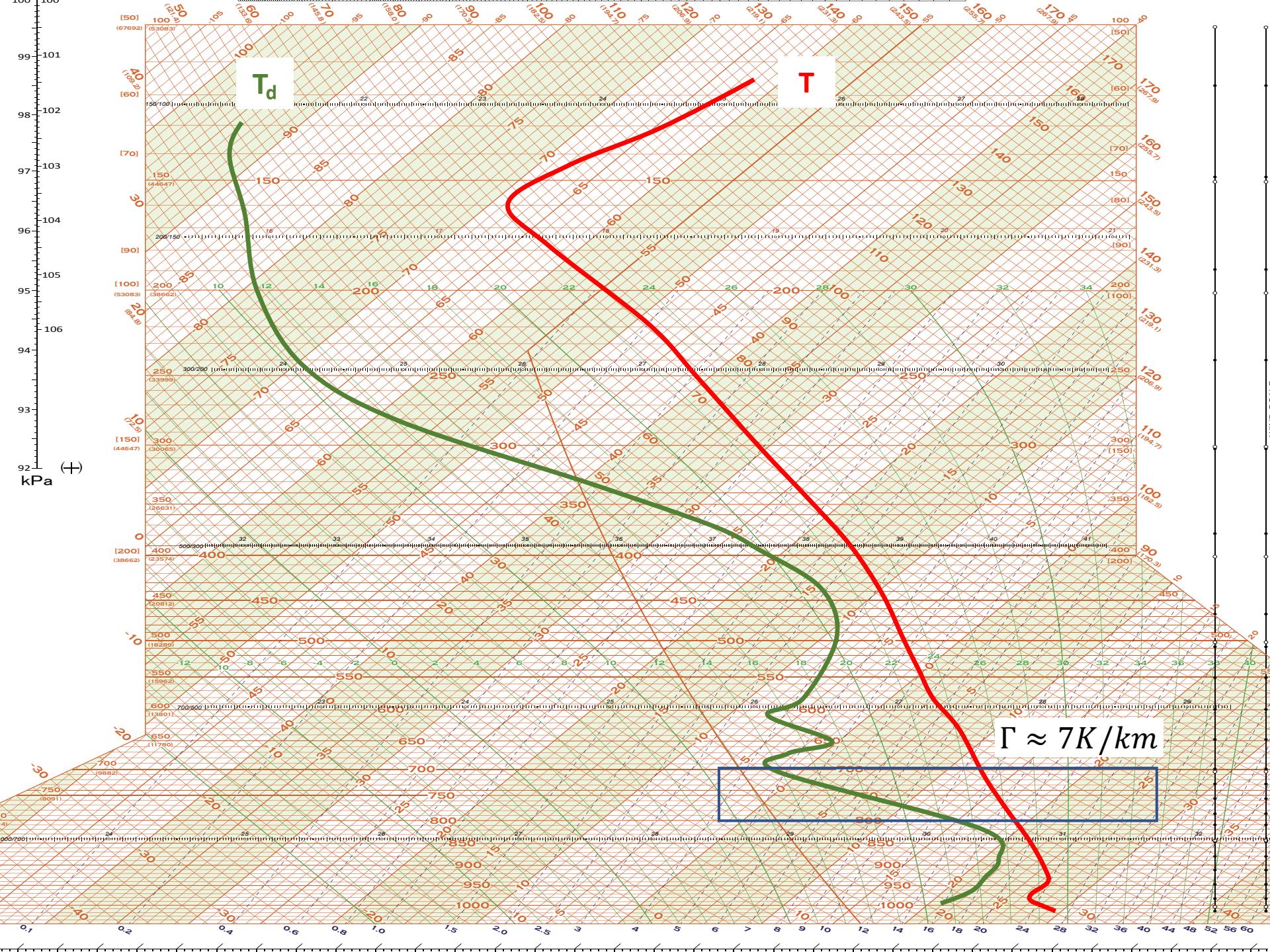
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Dry adiabatic lapse rate: Adiabatic lapse rate that applies to sub-saturated air: $\Gamma_d = \frac{g}{c_p} \approx 9.81\text{K/km}$

Moist adiabatic lapse rate: Adiabatic lapse rate that applies to saturated air:

$$\Gamma_m = g \frac{1 + \frac{L_v r}{RT}}{c_p + \frac{\epsilon L_v^2 r}{RT^2}}$$



Stability Metrics

Dry Air (Module 1.2)

| Temperature | Potential Temperature | Stability |
|---------------------|--|-----------|
| $\Gamma > \Gamma_d$ | $\frac{\partial \theta}{\partial z} < 0$ | Unstable |
| $\Gamma = \Gamma_d$ | $\frac{\partial \theta}{\partial z} = 0$ | Neutral |
| $\Gamma < \Gamma_d$ | $\frac{\partial \theta}{\partial z} > 0$ | Stable |

Moist air

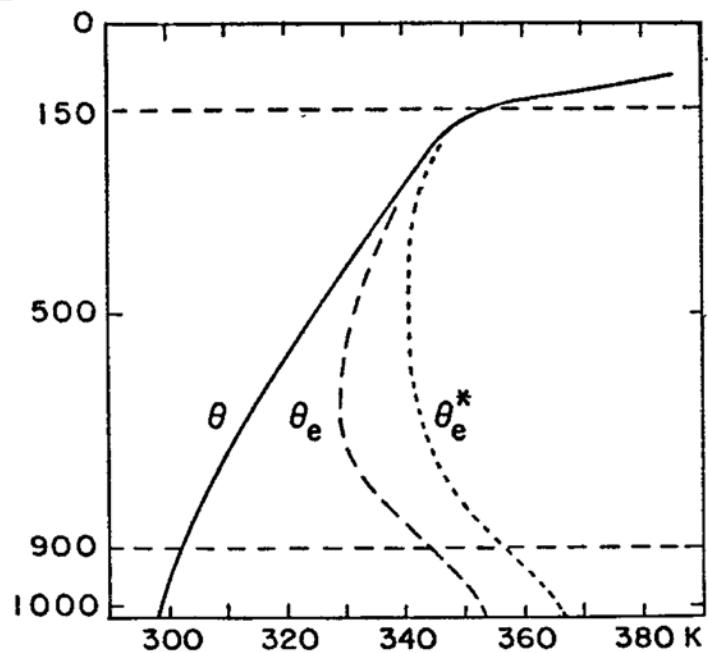
| Temperature | Saturated Equivalent Potential Temperature | Stability |
|--------------------------------------|---|------------------------|
| $\Gamma > \Gamma_d$ | $\frac{\partial \theta_{es}}{\partial z} < 0$ | Unstable |
| $\Gamma_m \leq \Gamma \leq \Gamma_d$ | $\frac{\partial \theta_{es}}{\partial z} < 0$ | Conditionally Unstable |
| $\Gamma = \Gamma_m$ | $\frac{\partial \theta_{es}}{\partial z} = 0$ | Moist Neutral |
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θ_e^* is just a different notation for θ_{es} .

Stability Metrics

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θ_e^* is the same as θ_{es} .

MR3252: Tropical Meteorology

Thermodynamic Profiles and Important Quantities in the Tropical Atmosphere, Part II

Main Topics:

- Interpretation of tropical thermodynamic profiles
- Moist static energy
- Convective available potential energy

Some quantities with which to be familiar:

Total Precipitable Water (TPW): The depth of a hypothetical unit volume of liquid water that would be produced if all water vapor in the atmosphere were precipitated out

$$TPW = \frac{\int_{PSFC}^0 r(p) dp}{\rho g}$$

Moist static energy (MSE, often h): Similar to θ_e .

Sort of conserved in adiabatic processes, but not exactly if we consider hydrometeors in q .

$$h \approx c_p T + \Phi + L_v q$$

Brackets often used to indicate “column-integrated” values. Here s is dry static energy.

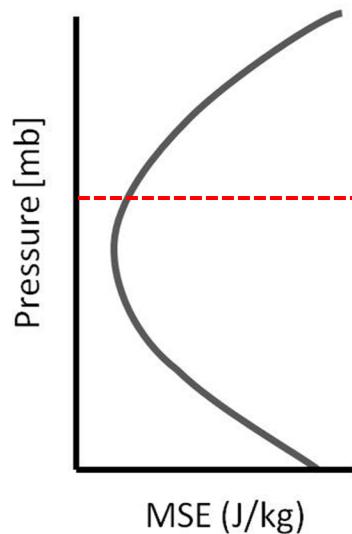
Gross moist stability (GMS, many variables used, M here): A quantity that compares the lateral export of some conserved quantity (in adiabatic processes) to some measure of convective intensity. GMS is negative if MSE is imported (in net) into a column. See Raymond et al. (2009) and Inoue and Back (2015).

$$M = \frac{\nabla \cdot \langle h \mathbf{v} \rangle}{\nabla \cdot \langle s \mathbf{v} \rangle}$$

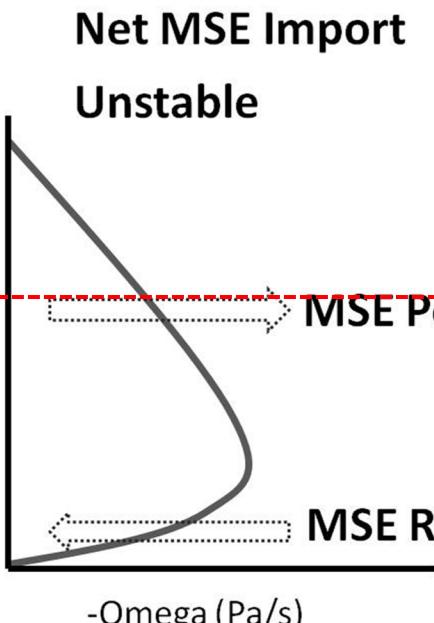
Convective available potential energy (CAPE): Maximum buoyancy of *undiluted* air parcel.

$$CAPE = \int_{P_{LFC}}^{P_{EL}} R(T_{v,p} - T_{v,e}) d \ln p$$

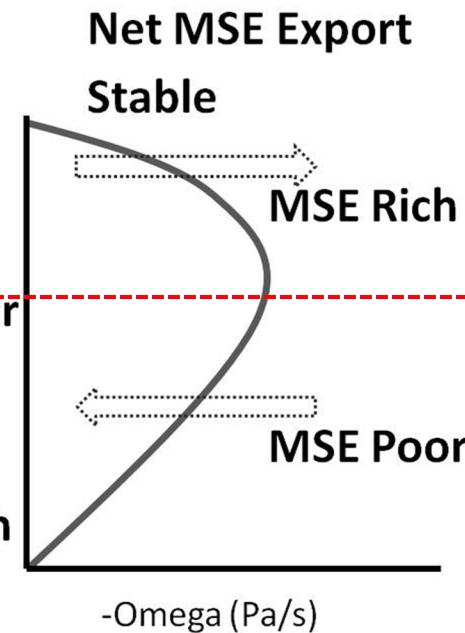
Typical MSE profile



Negative GMS



Positive GMS



$\sim 500 \text{ hPa}$

Inoue and Back (2015)

Some quantities with which to be familiar:

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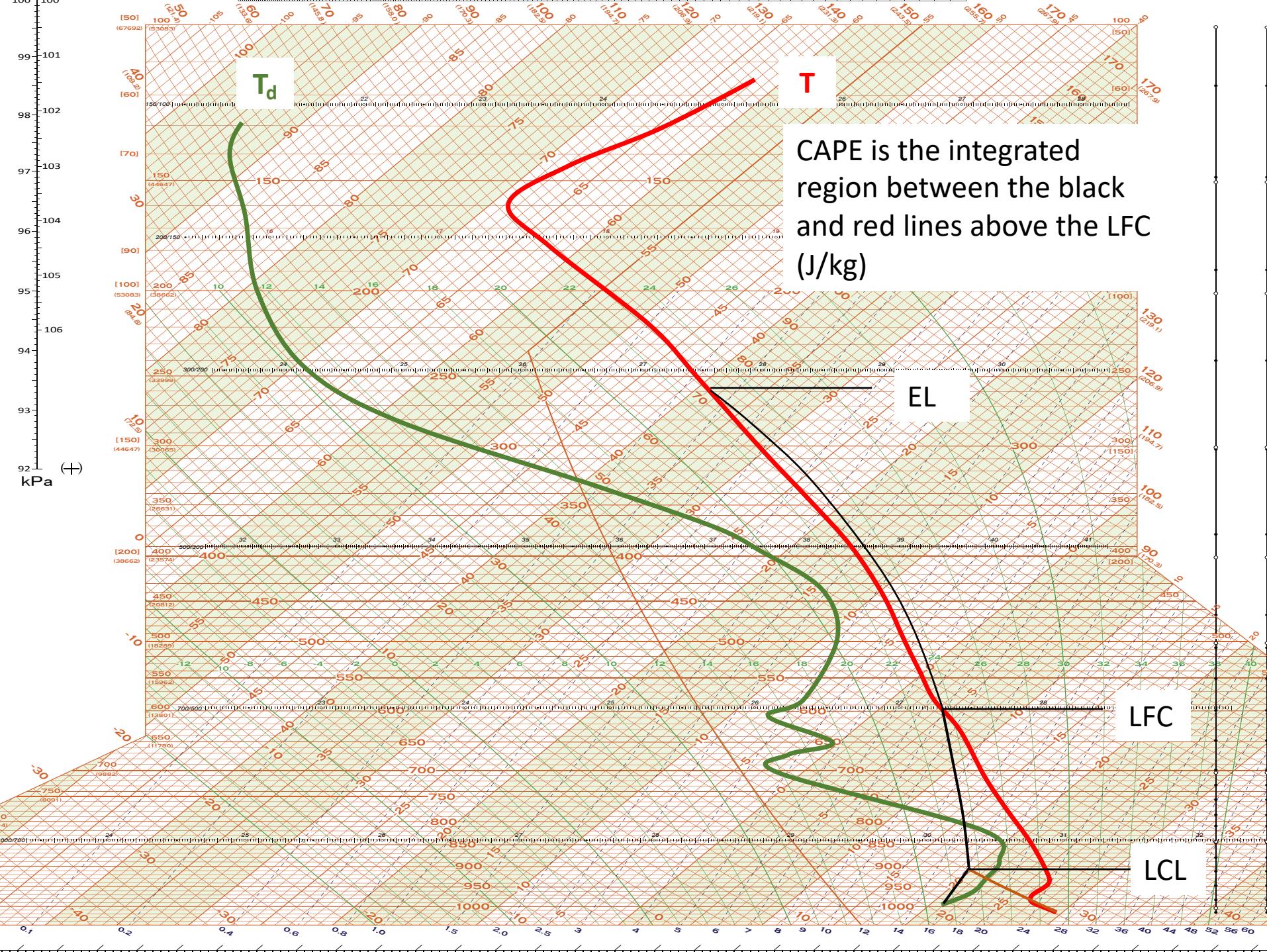
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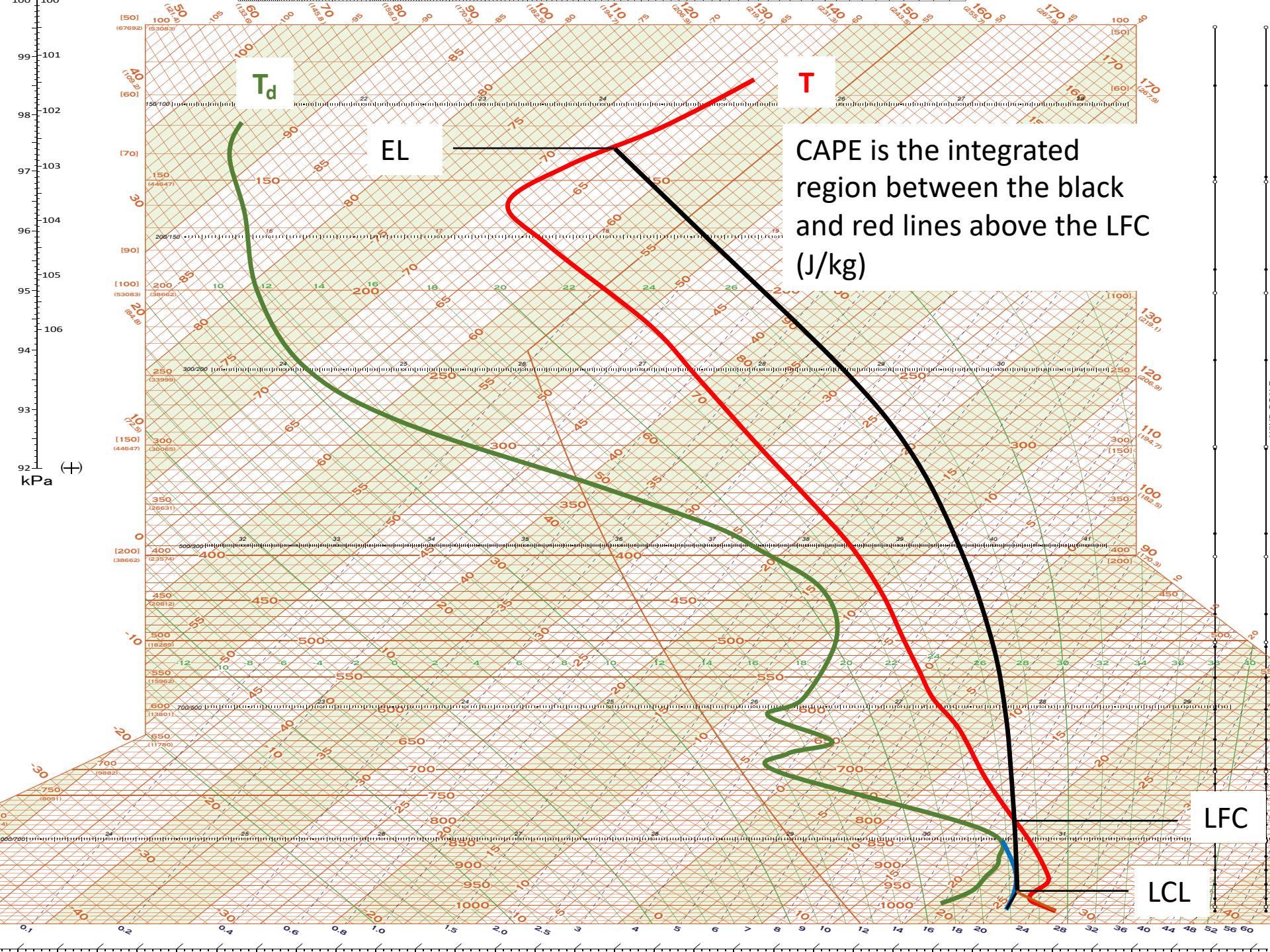
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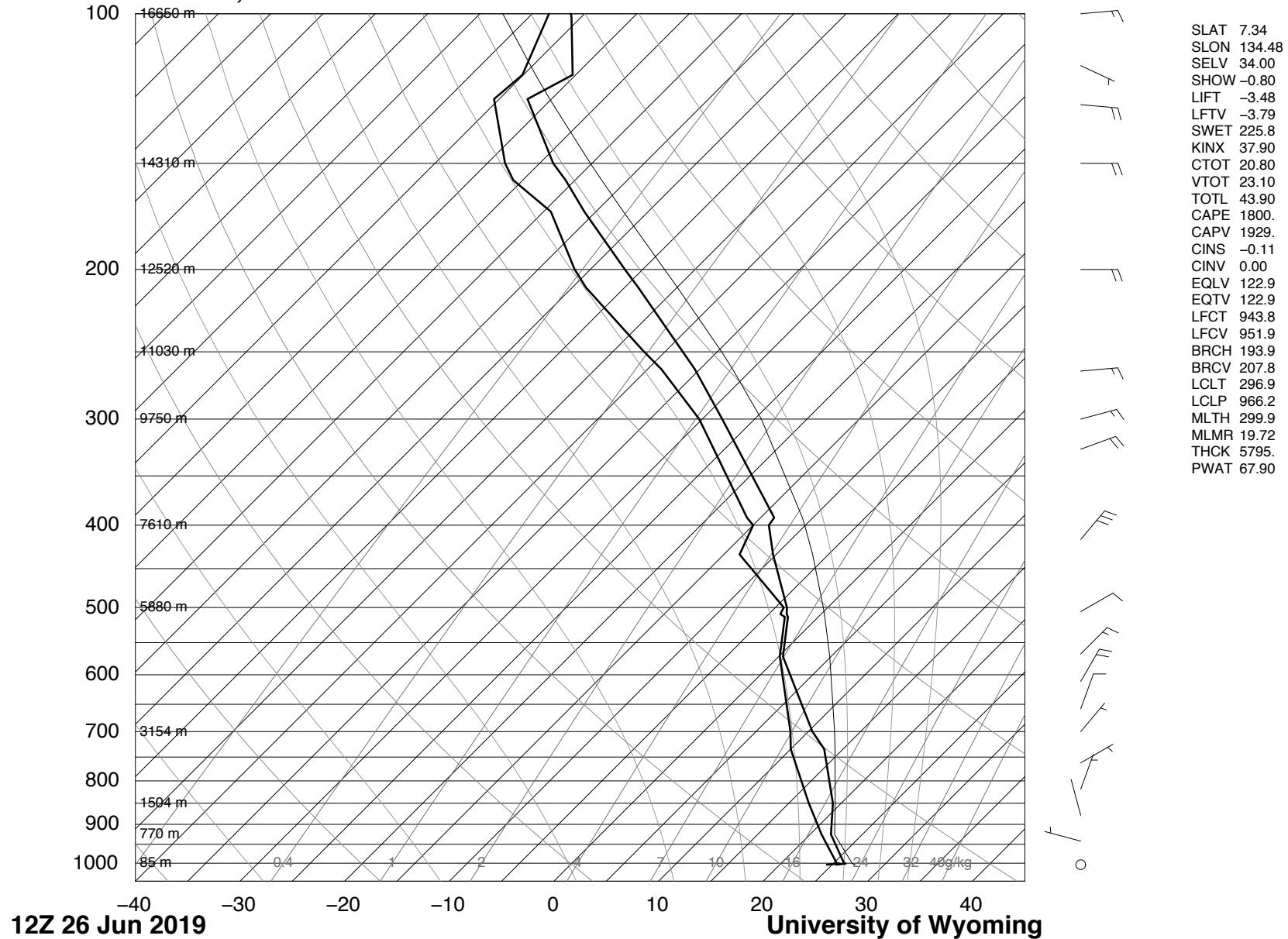


CAPE is the integrated
region between the black
and red lines above the LFC
(J/kg)



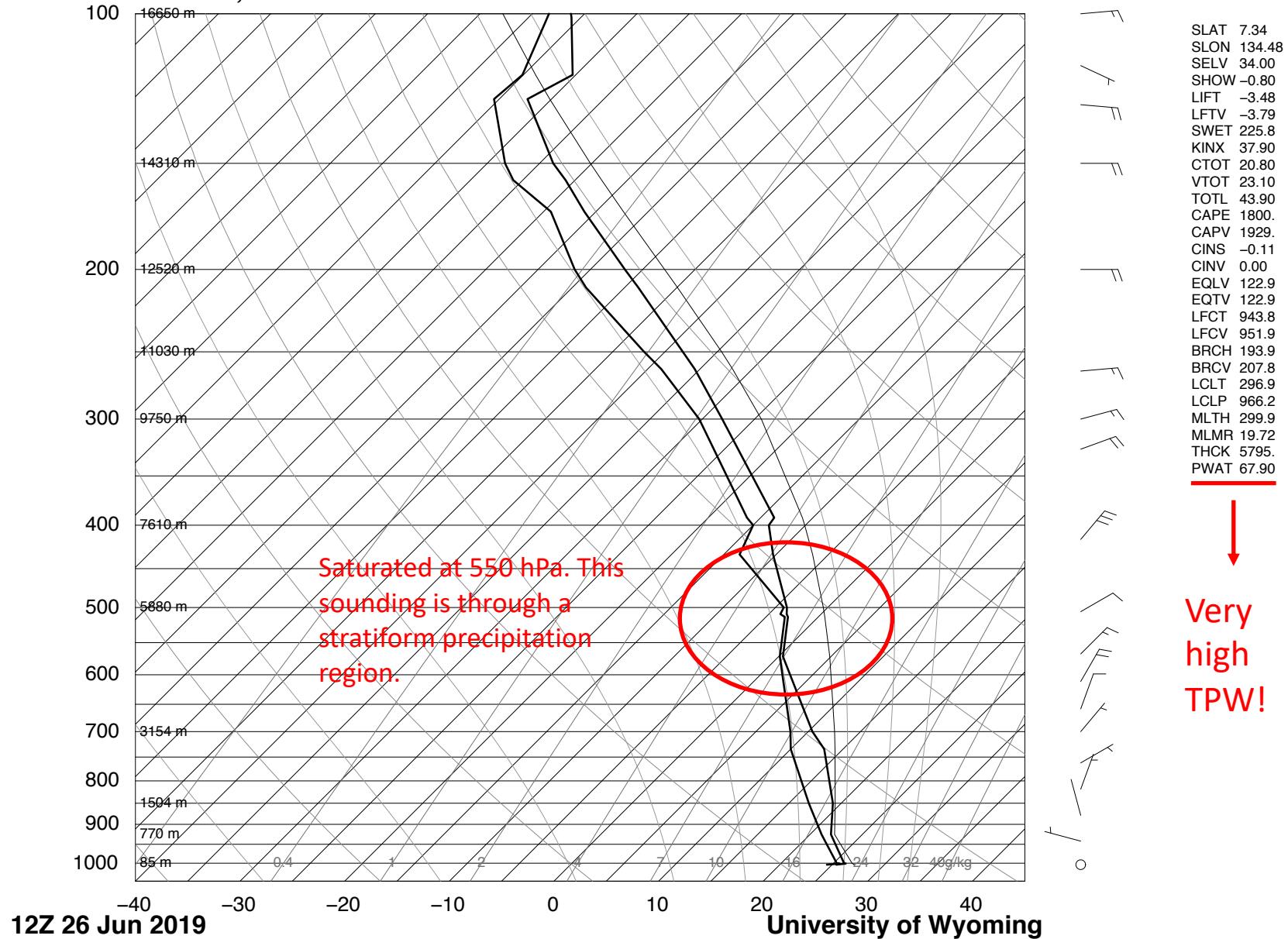
Tropical West Pacific: Near-saturated sounding, probably through organized MCS

91408 PTRO Koror, Palau Is



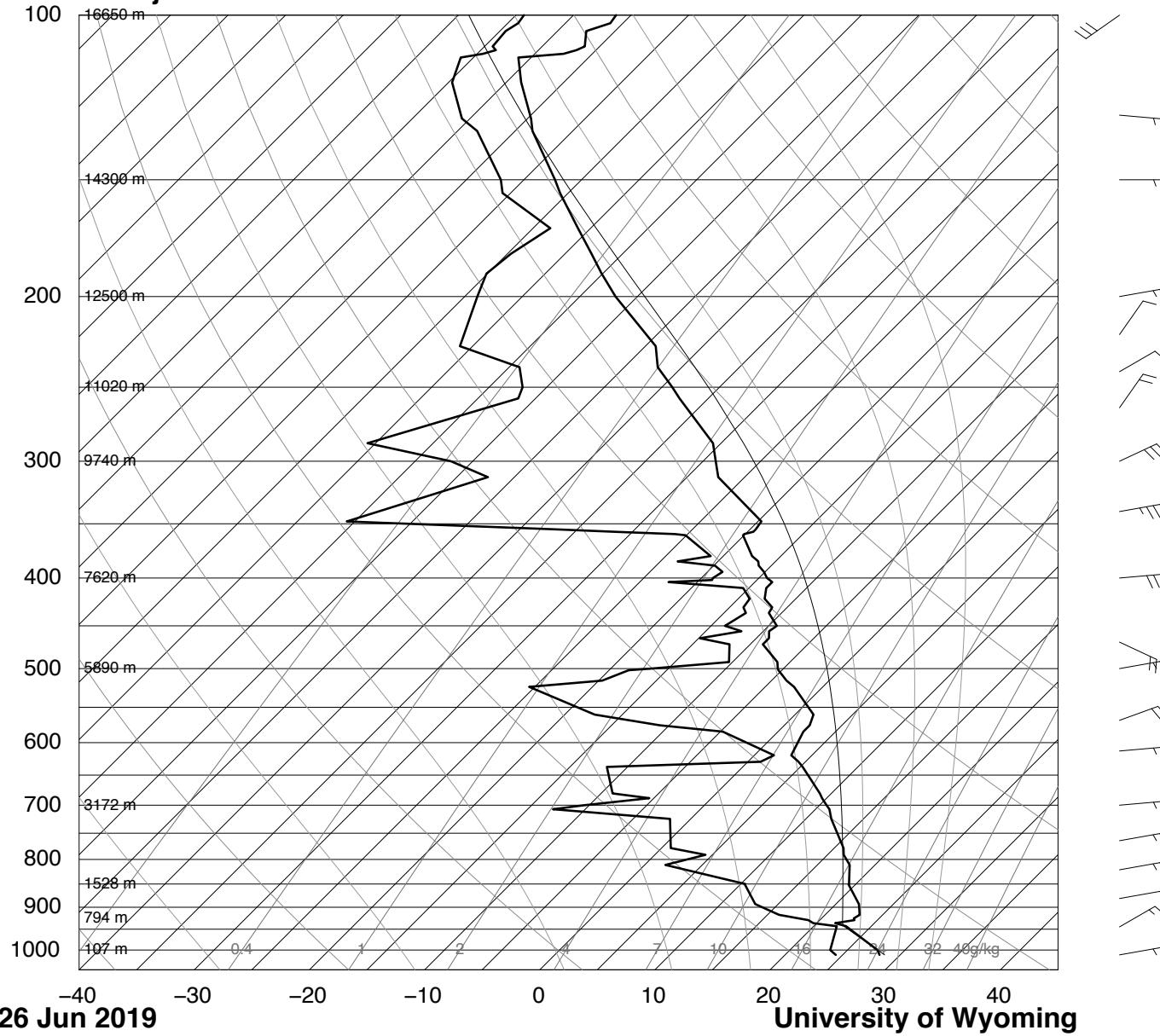
Tropical West Pacific: Near-saturated sounding, probably through organized MCS

91408 PTRO Koror, Palau Is



Tropical West Pacific: Drier marine sounding; supportive of shallow convection, but not deep, organized convection

91376 PKMJ Majuro

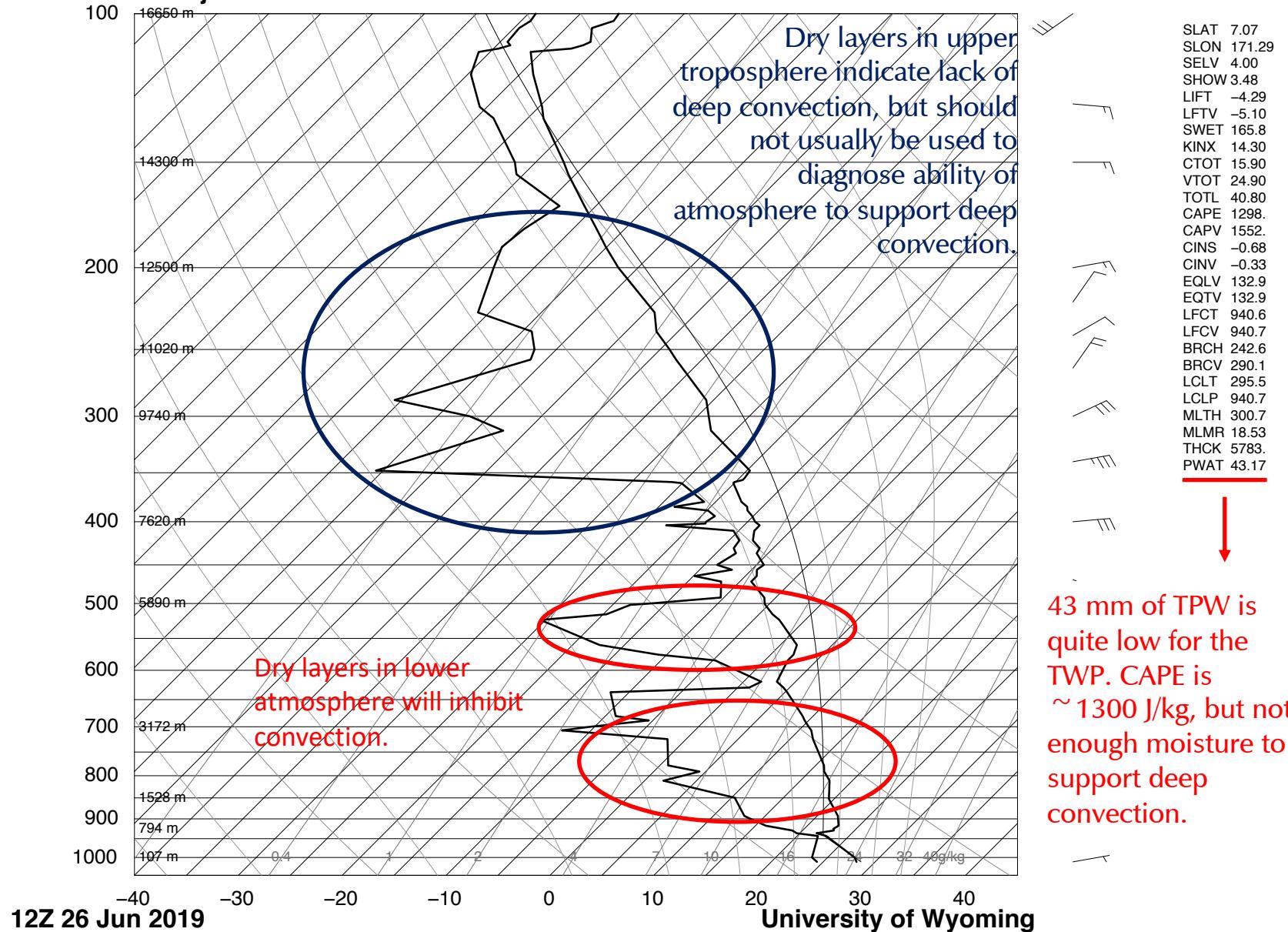


12Z 26 Jun 2019

University of Wyoming

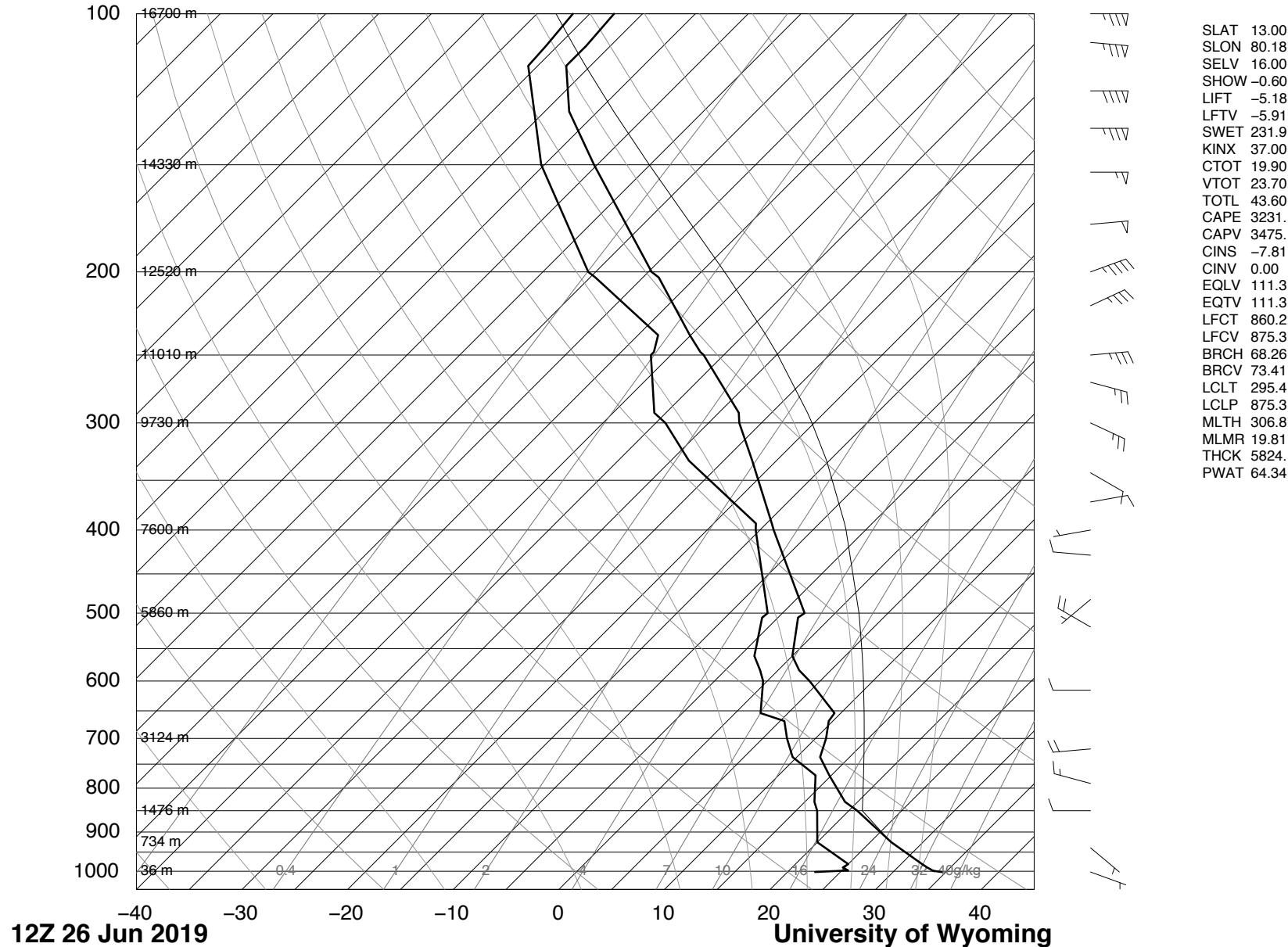
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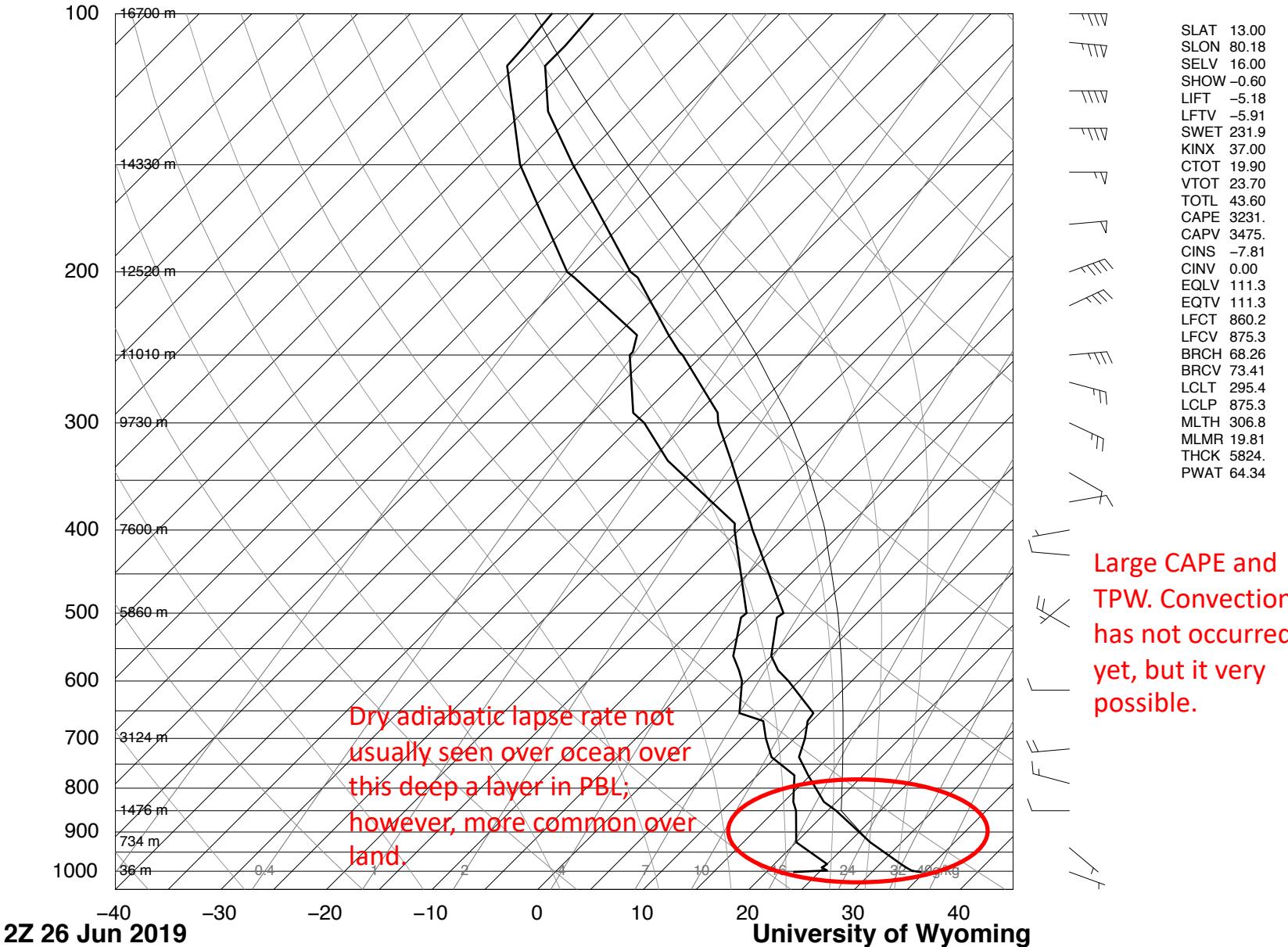
South Asia coastal sounding: Lots of moisture and CAPE. PBL lapse rate almost dry adiabatic. High LCL/LFC though, forcing needed for deep convection.

43279 VOMM Madras



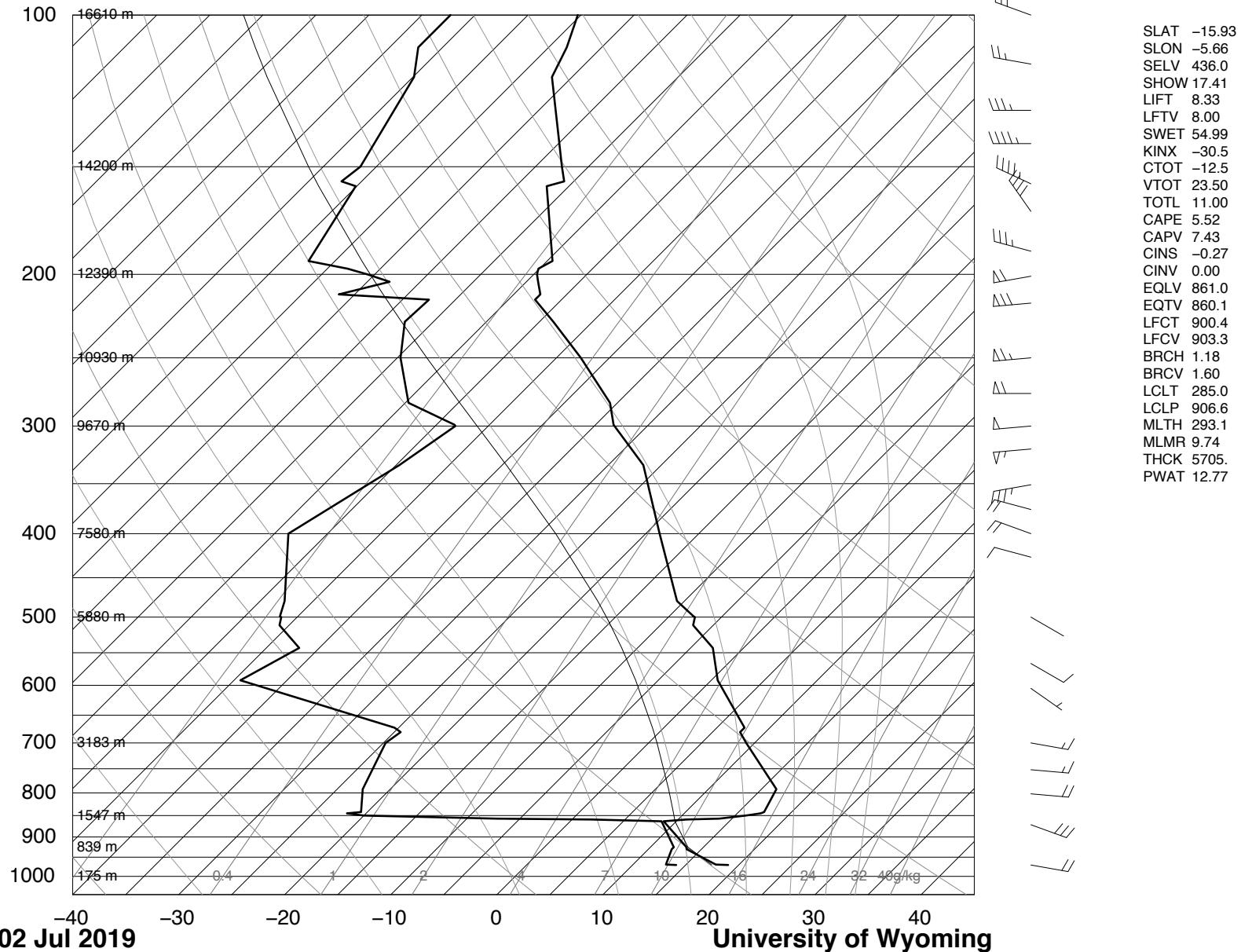
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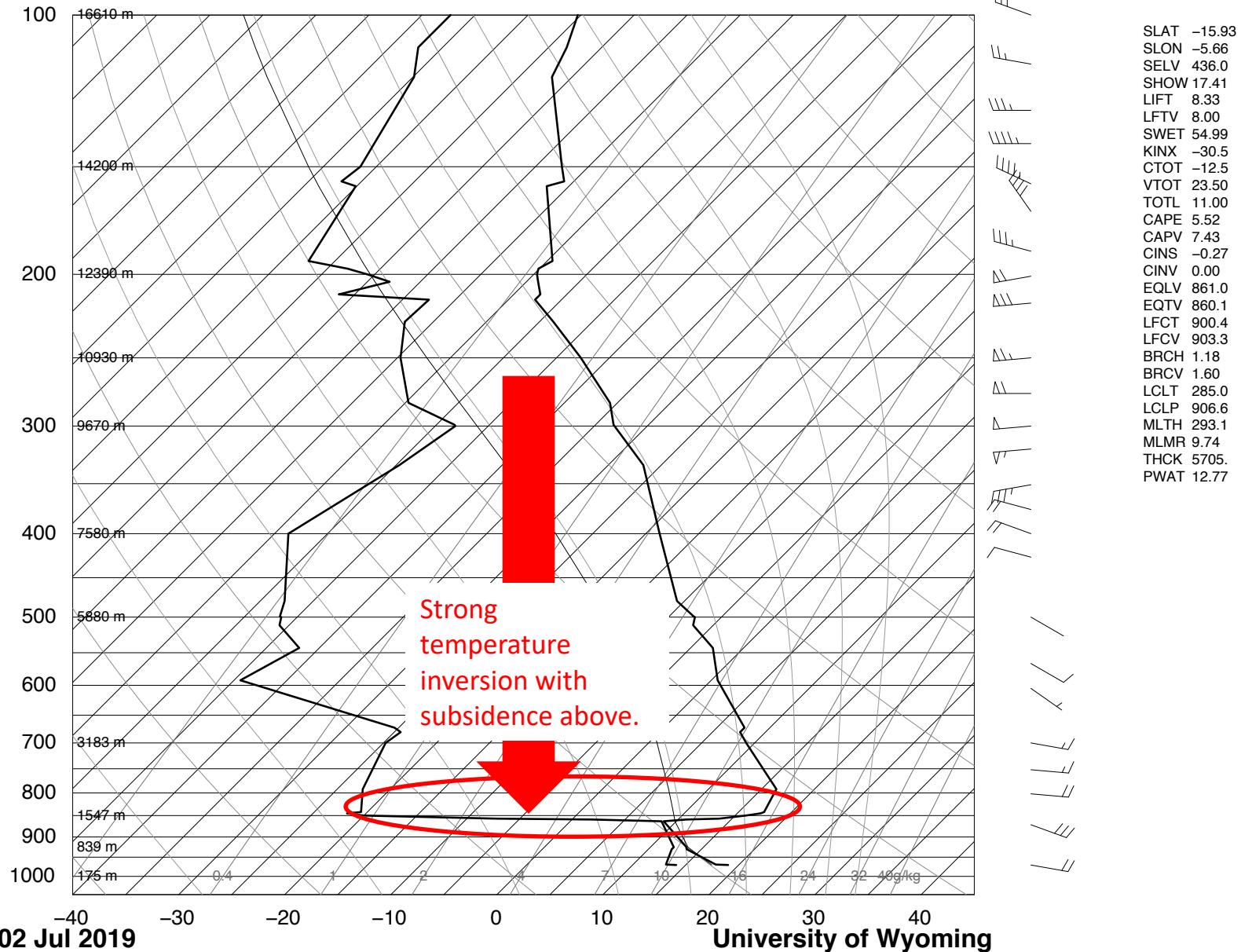
Tropical East Atlantic sounding: Strong inversion and subsidence drying above PBL; stratocumulus common

61901 St. Helena Is.



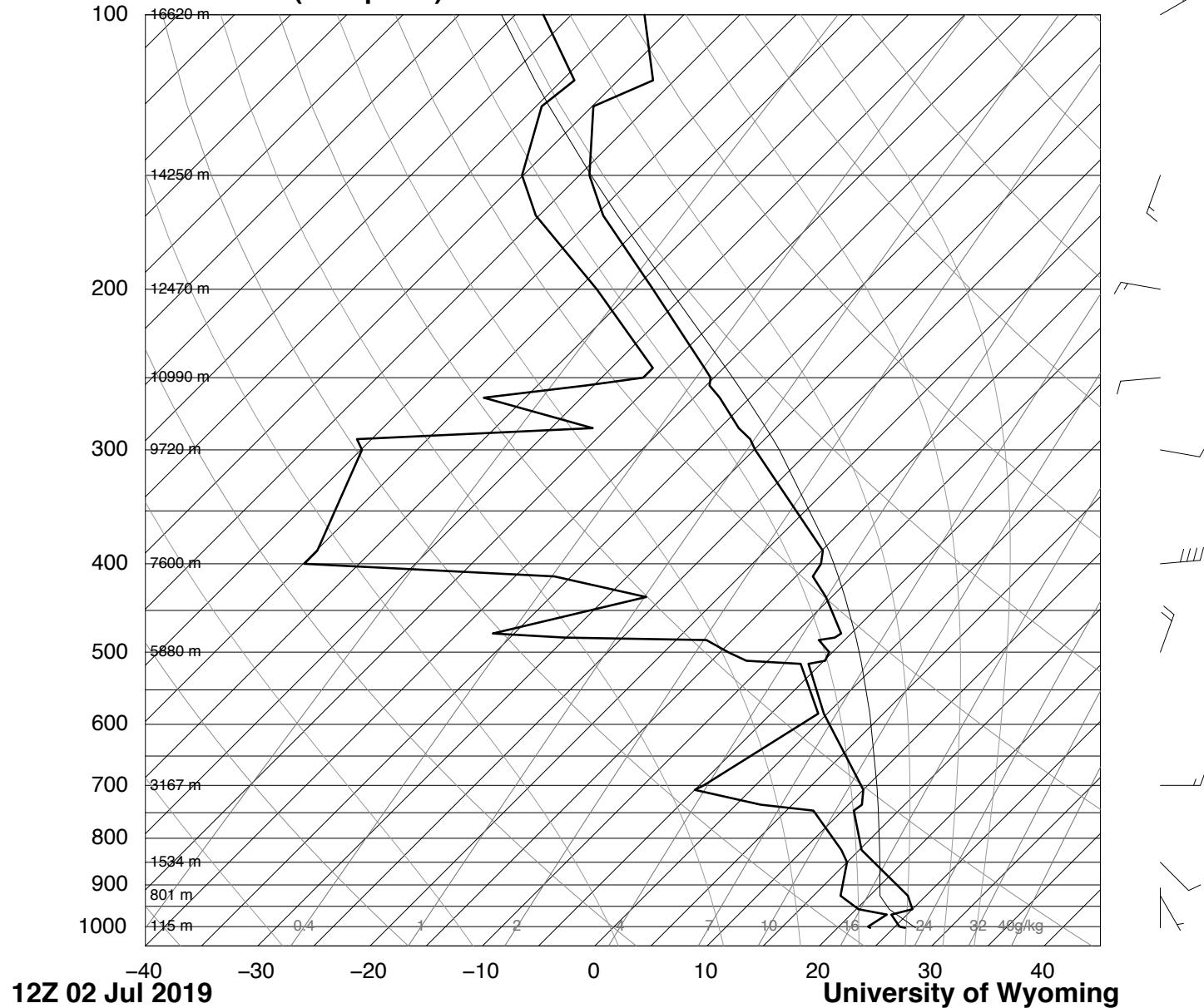
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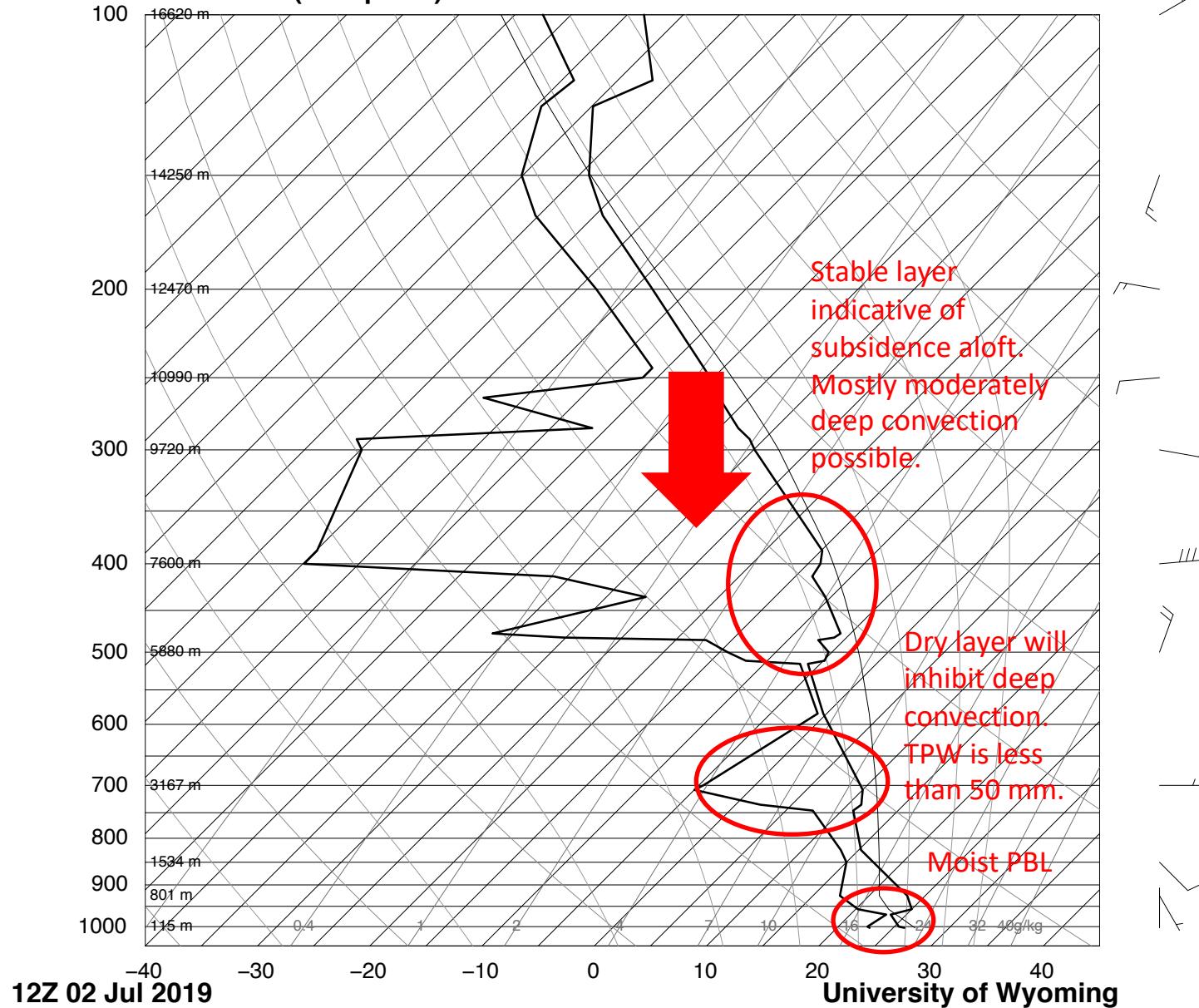
Amazonian sounding: Can be much moister than what is shown below.
 Rainforest acts like “green ocean” to keep PBL moist.

82332 SBMN Manaus (Aeroporto)



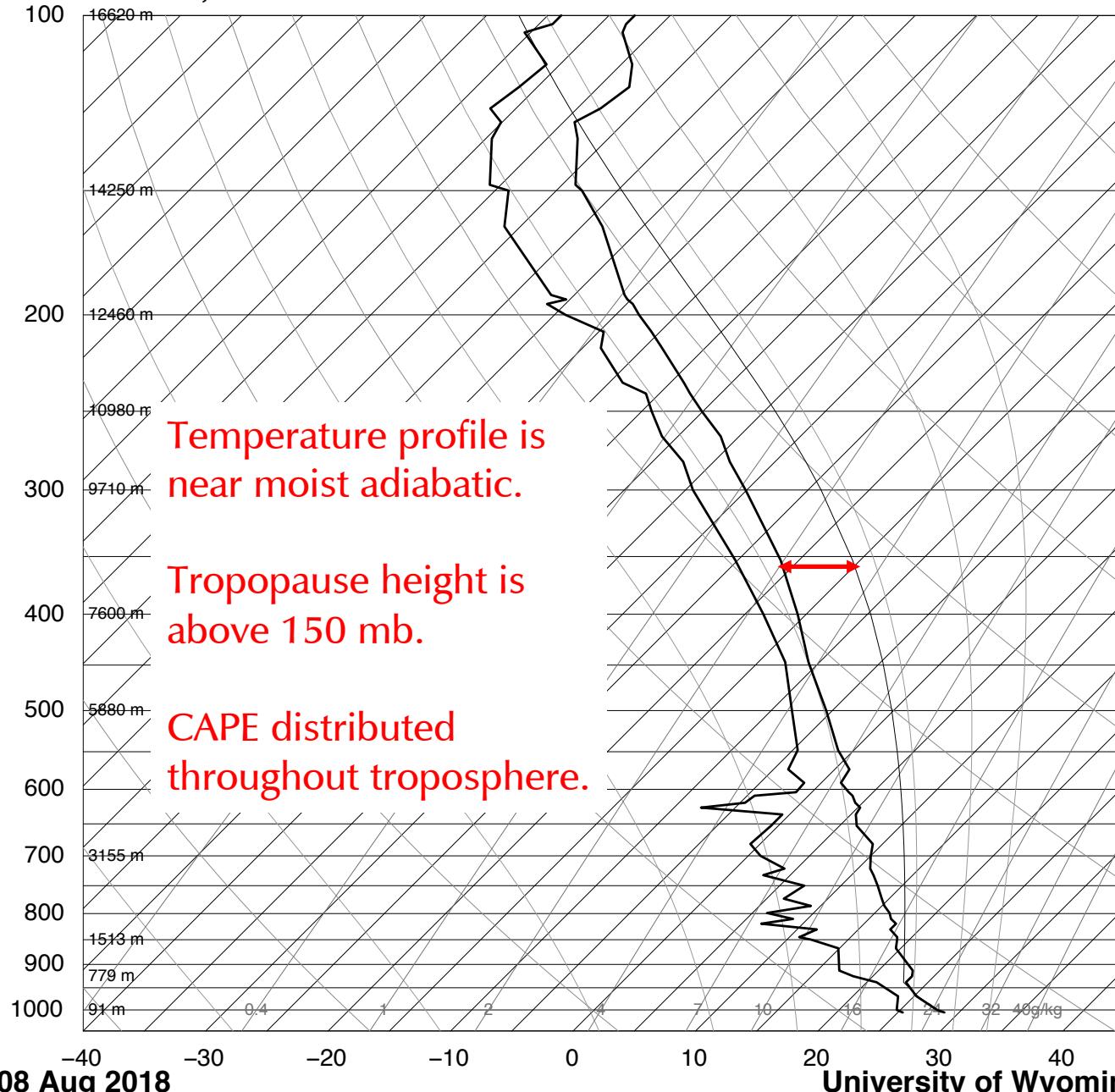
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Compare two soundings with similar CAPE

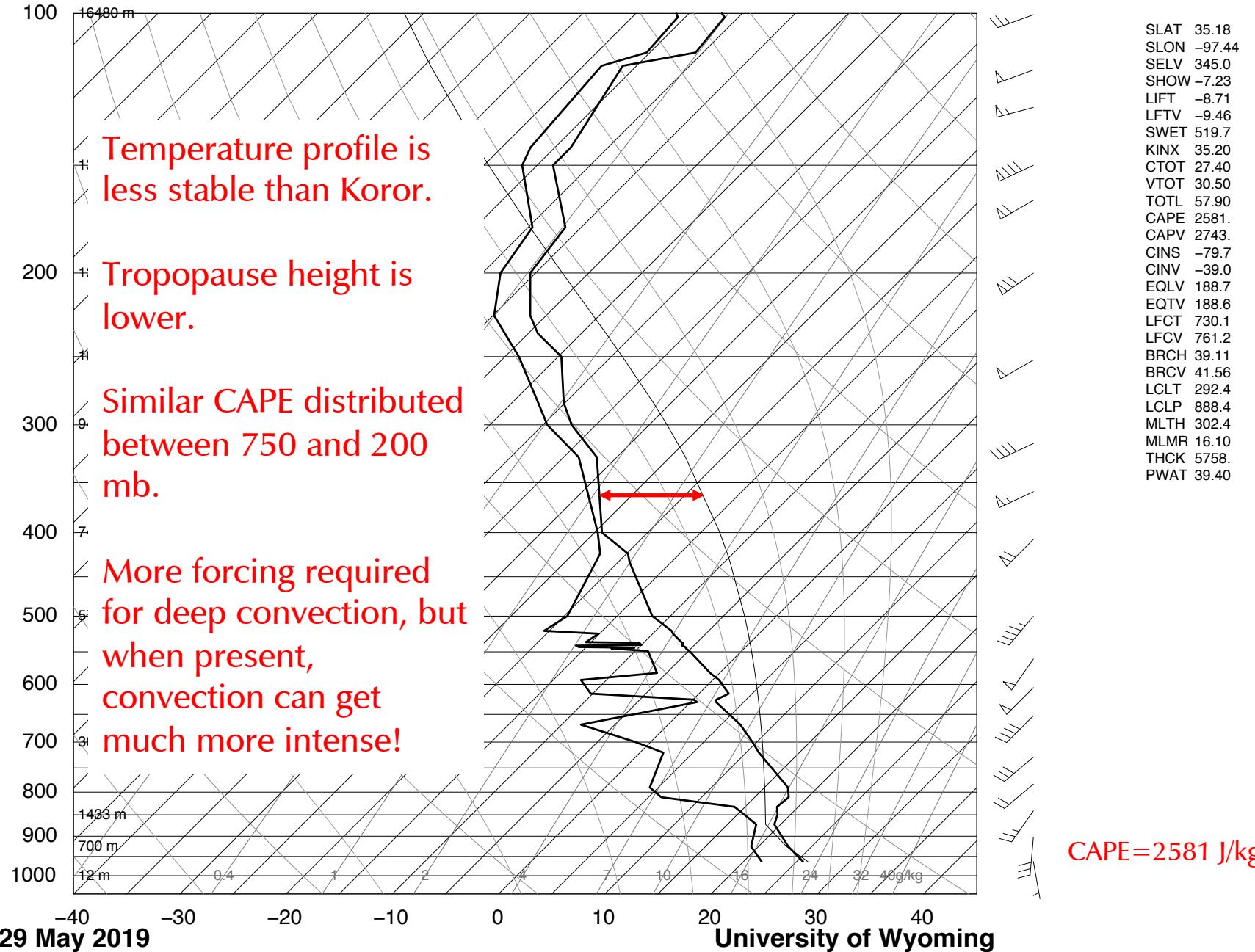
91408 PTRO Koror, Palau Is



| | |
|------|--------|
| SLAT | 7.34 |
| SLON | 134.48 |
| SELV | 34.00 |
| SHOW | 2.32 |
| LIFT | -5.36 |
| LFTV | -5.96 |
| SWET | 195.0 |
| KINX | 27.50 |
| CTOT | 17.50 |
| VTOT | 24.50 |
| TOTL | 42.00 |
| CAPE | 2530. |
| CAPV | 2767. |
| CINS | -8.18 |
| CINV | -0.02 |
| EQLV | 127.5 |
| EQTV | 127.4 |
| LFCT | 889.4 |
| LFCV | 944.1 |
| BRCH | 399.7 |
| BRCV | 437.2 |
| LCLT | 296.4 |
| LCLP | 946.3 |
| MLTH | 301.1 |
| MLMR | 19.49 |
| THCK | 5789. |
| PWAT | 52.97 |

Compare two soundings with similar CAPE

72357 OUN Norman

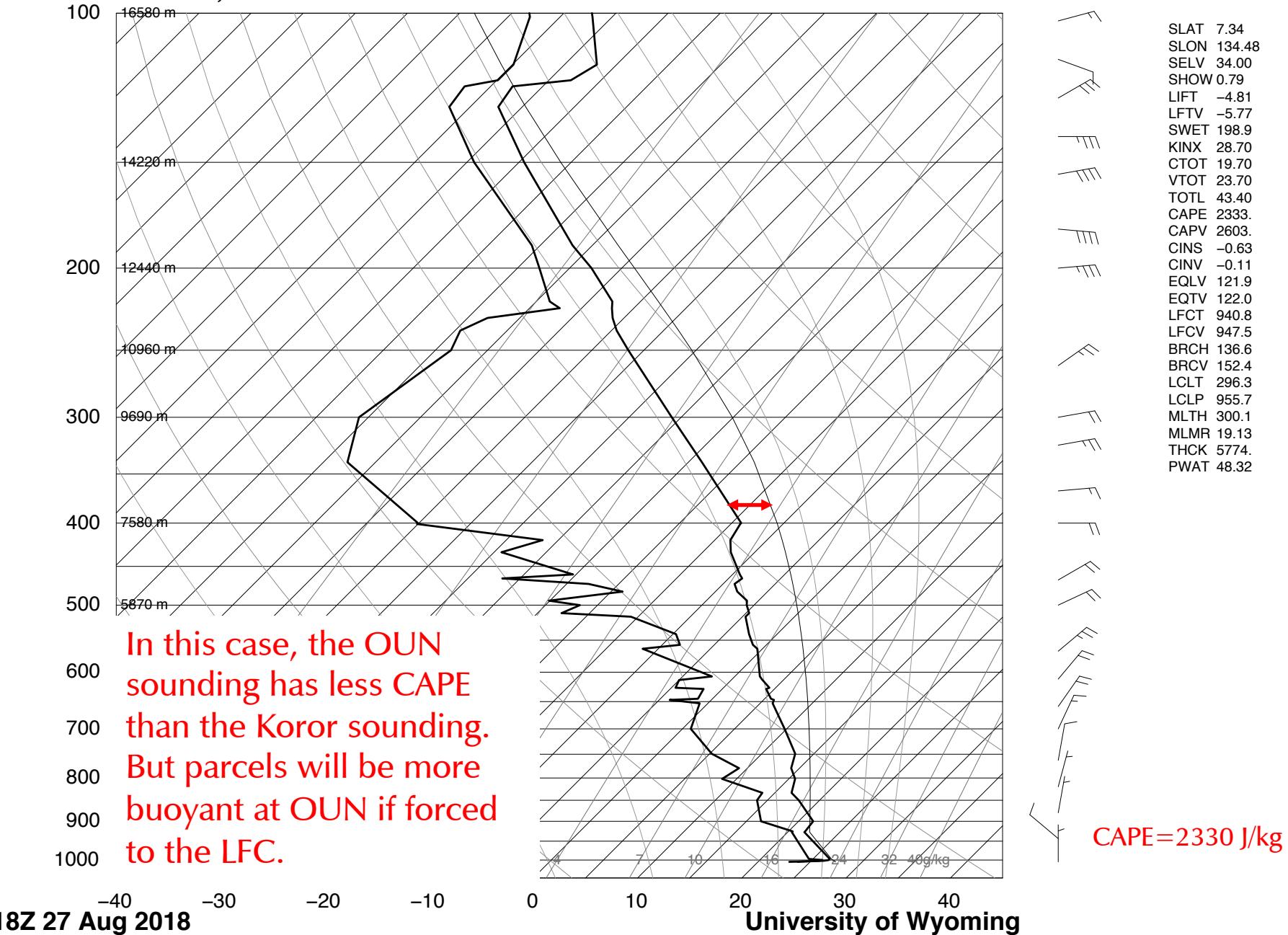


00Z 29 May 2019

University of Wyoming

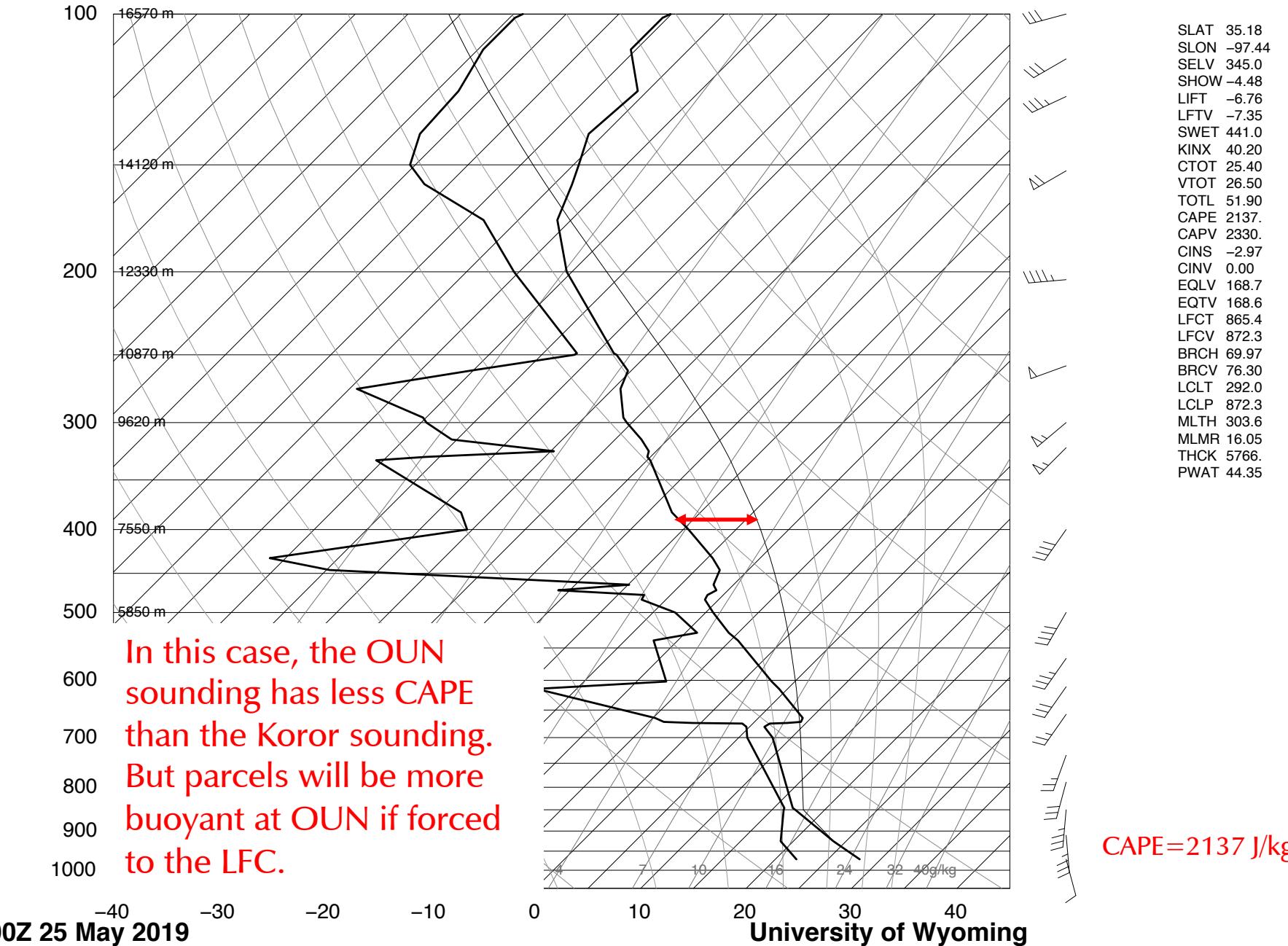
Another Example

91408 PTRO Koror, Palau Is



Another Example

72357 OUN Norman



MR3252: Tropical Meteorology

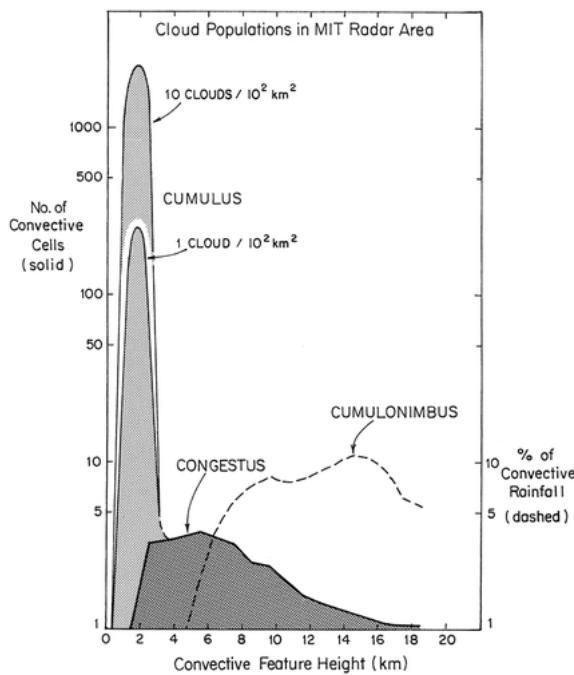
Modes of Cumuliform Convection

Main Topics:

- Shallow cumulus, congestus, deep convection
- Radiative-convective equilibrium
- Large-scale subsidence

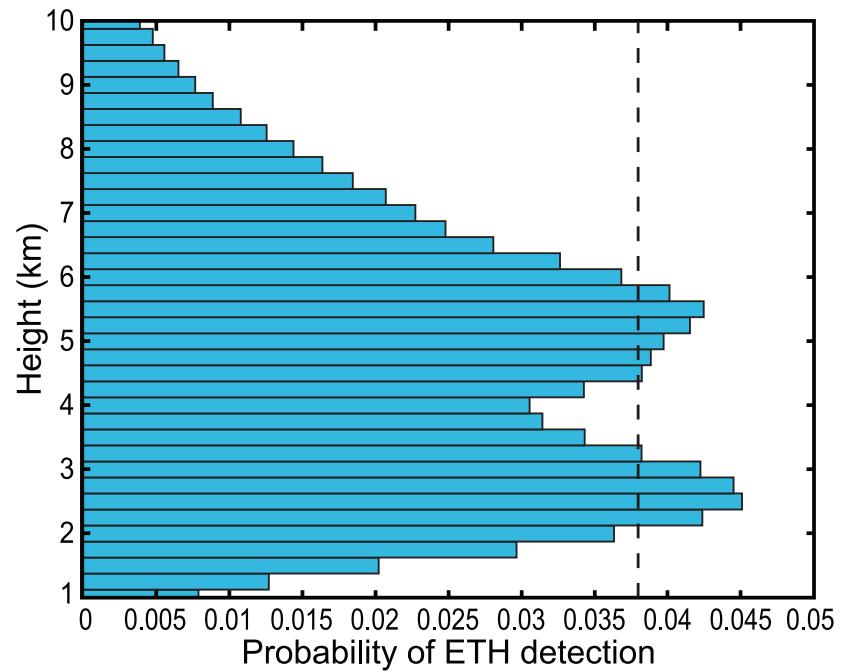
Trimodal distribution of convection

Radar data during TOGA COARE



Johnson et al.
(1999)

In TRMM data over TWP: Distribution of 20 dBZ Echo Top Heights (Moderate and Deep modes visible)



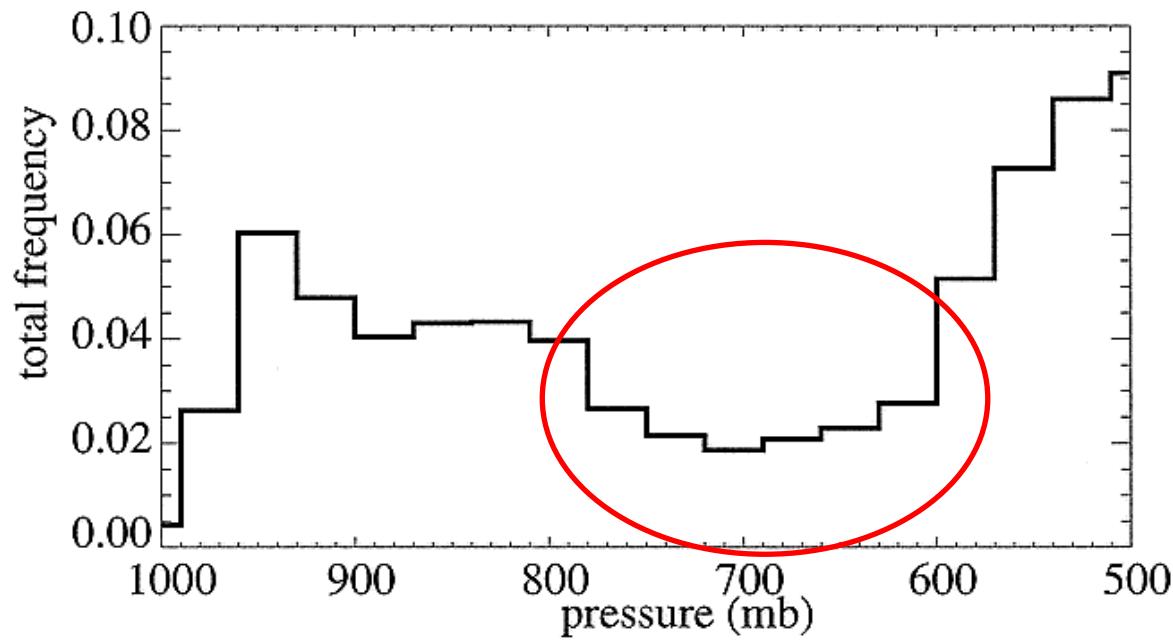
Powell and Houze
(2015)



Shallow convection

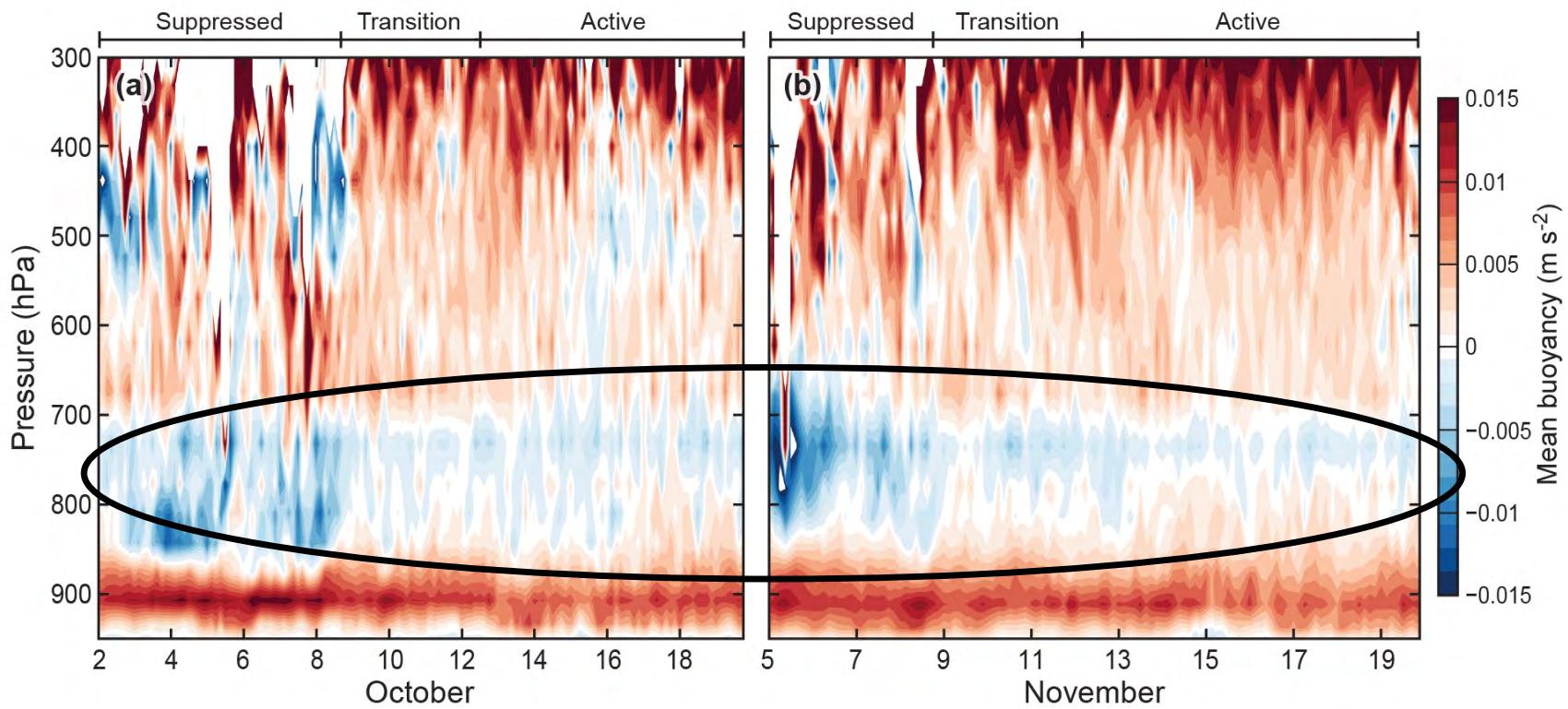
- Common when free troposphere is dry
- In some locations, often seen with large subsidence signal aloft, indicating an inversion often seen in the trade wind regime. Thus, sometimes these are called “trade cumuli”

Zuidema (1998): estimated using soundings



Minimum of clouds above boundary layer

Powell (2016): model of warm pool convection

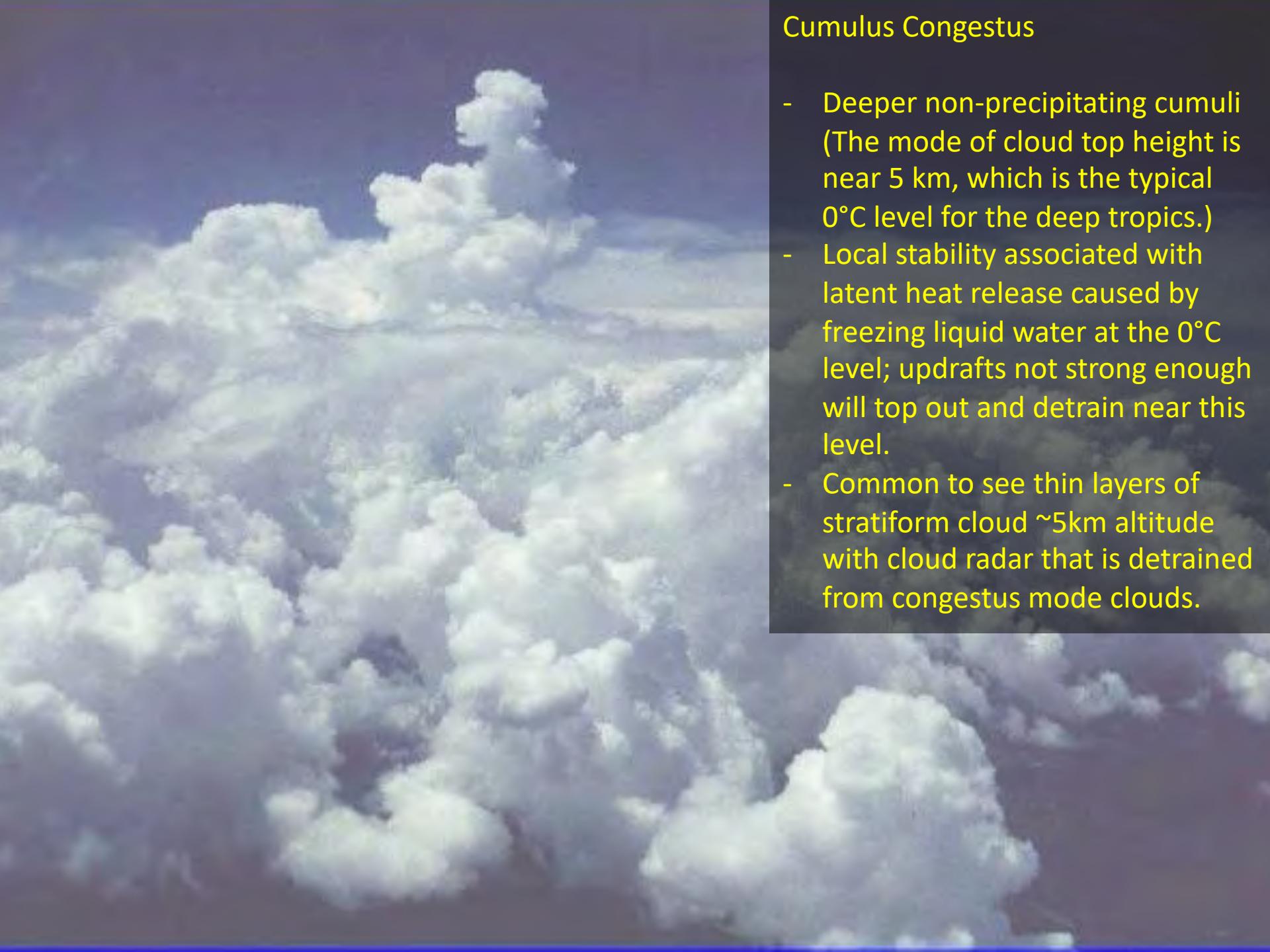


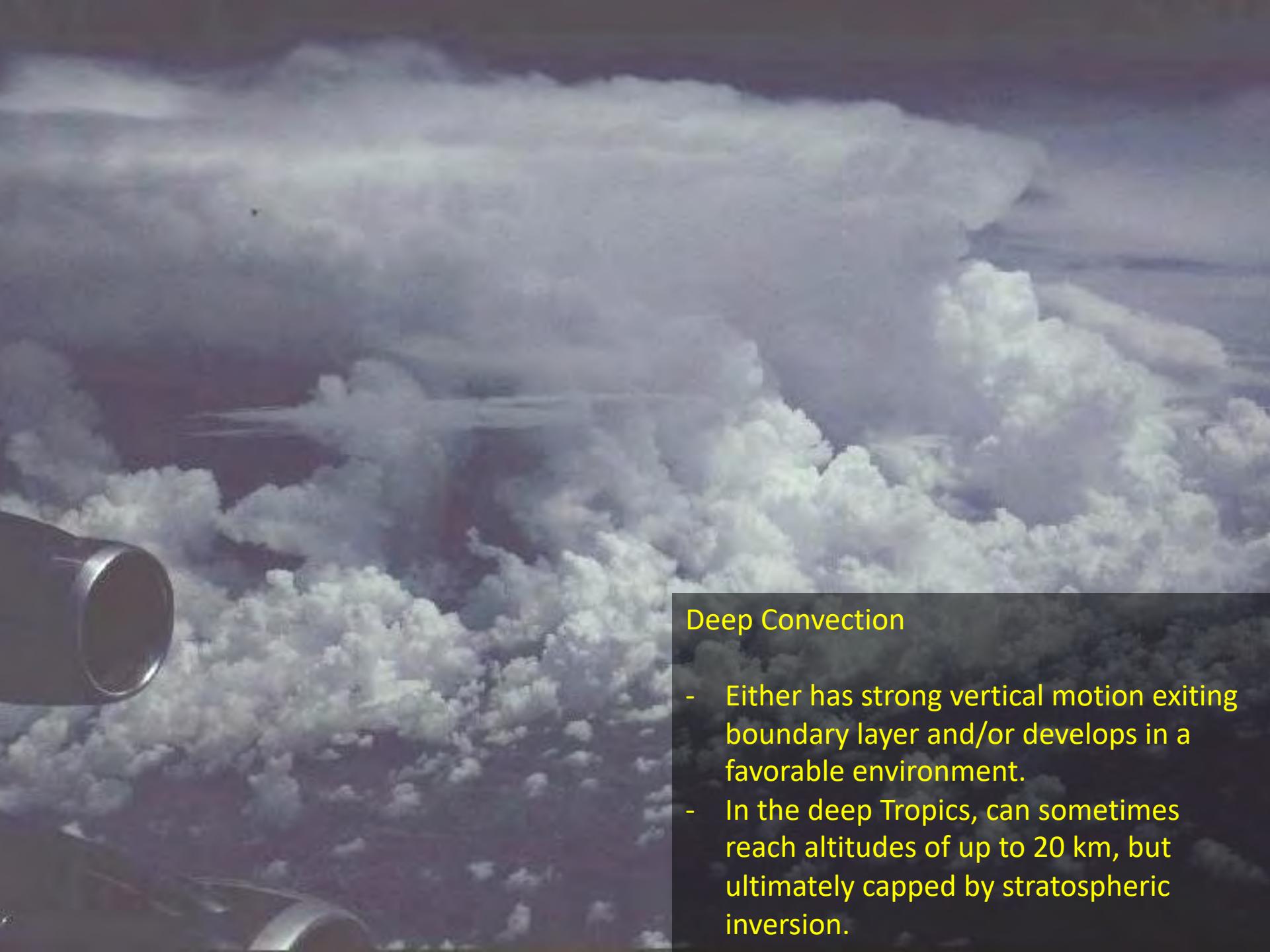
Negative buoyancy typically present between about 700 and 850 mb.

$$B \approx g \left(\underbrace{\frac{T^*}{T_e}}_{Temperature} - \underbrace{\frac{p^*}{p_e}}_{\substack{Pressure \\ Vapor}} + \underbrace{0.608(w^*)}_{Vapor} - \underbrace{w_H}_{Hydrometeor} \right)$$

Cumulus Congestus

- Deeper non-precipitating cumuli
(The mode of cloud top height is near 5 km, which is the typical 0°C level for the deep tropics.)
- Local stability associated with latent heat release caused by freezing liquid water at the 0°C level; updrafts not strong enough will top out and detrain near this level.
- Common to see thin layers of stratiform cloud ~5km altitude with cloud radar that is detrained from congestus mode clouds.

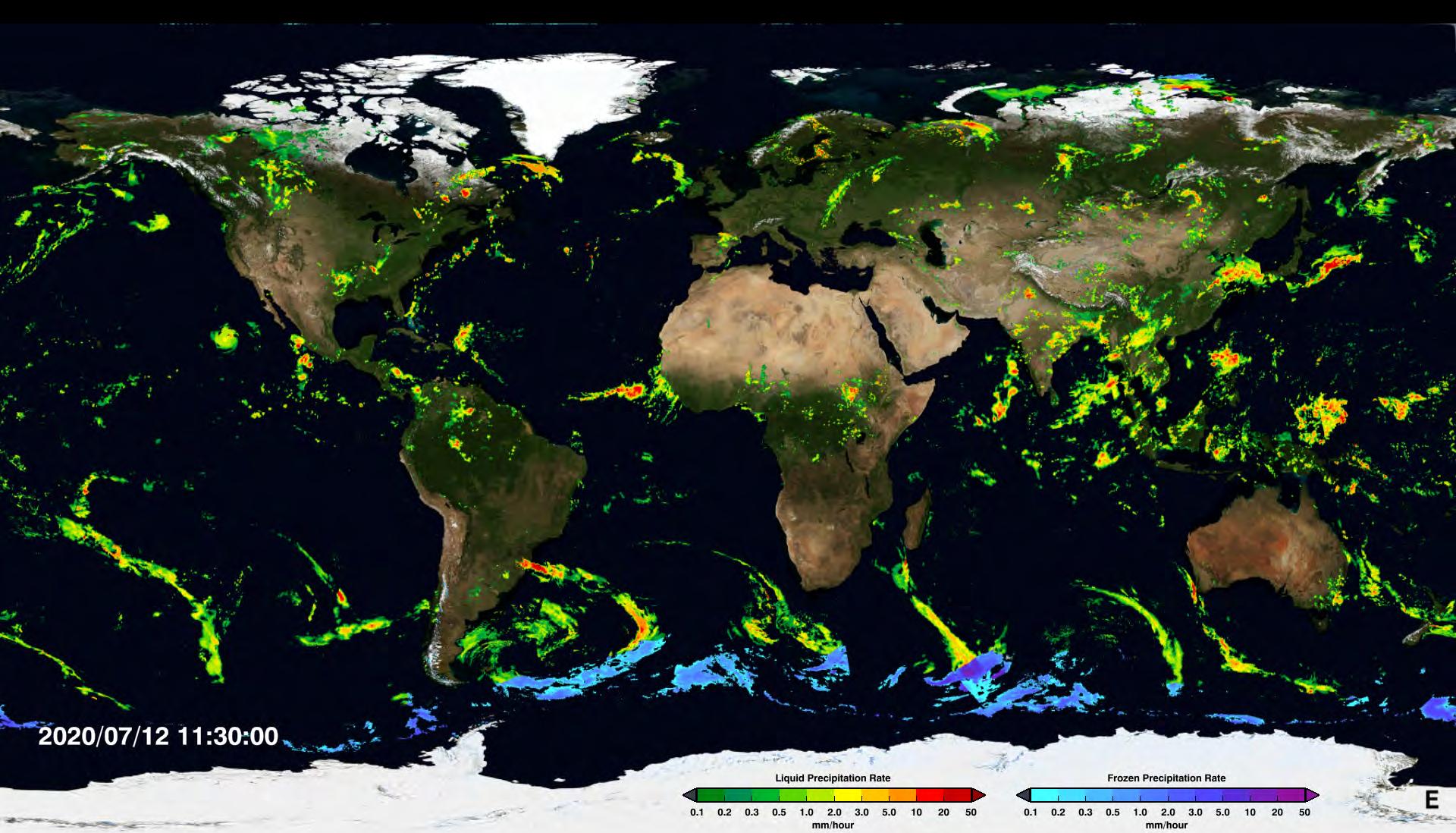




Deep Convection

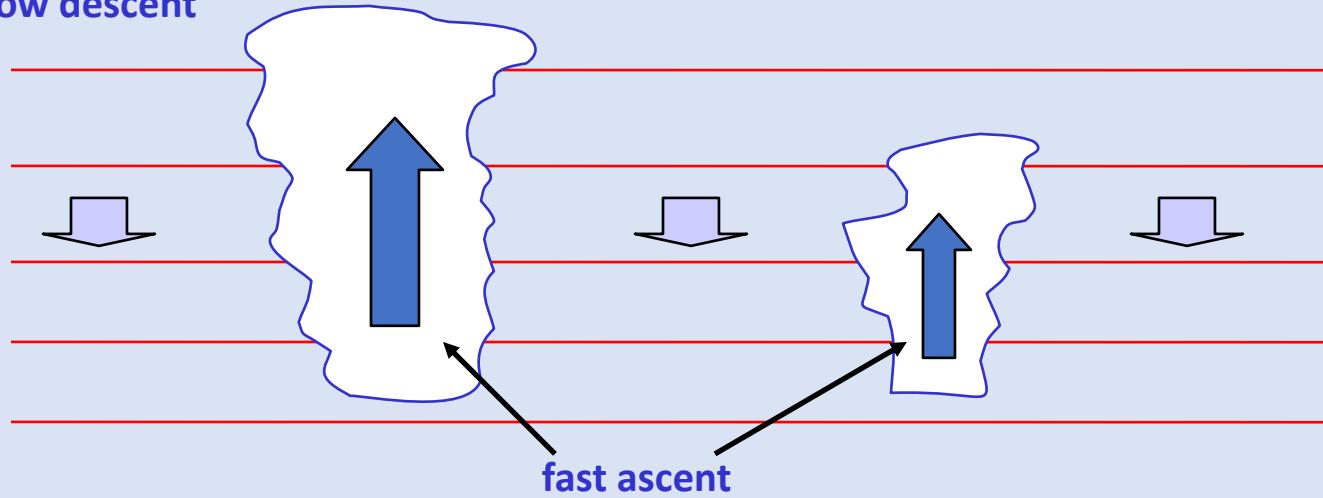
- Either has strong vertical motion exiting boundary layer and/or develops in a favorable environment.
- In the deep Tropics, can sometimes reach altitudes of up to 20 km, but ultimately capped by stratospheric inversion.

At any given time, rain occurs only over a small area!



Subsidence occurs in clear-air outside clouds

slow descent

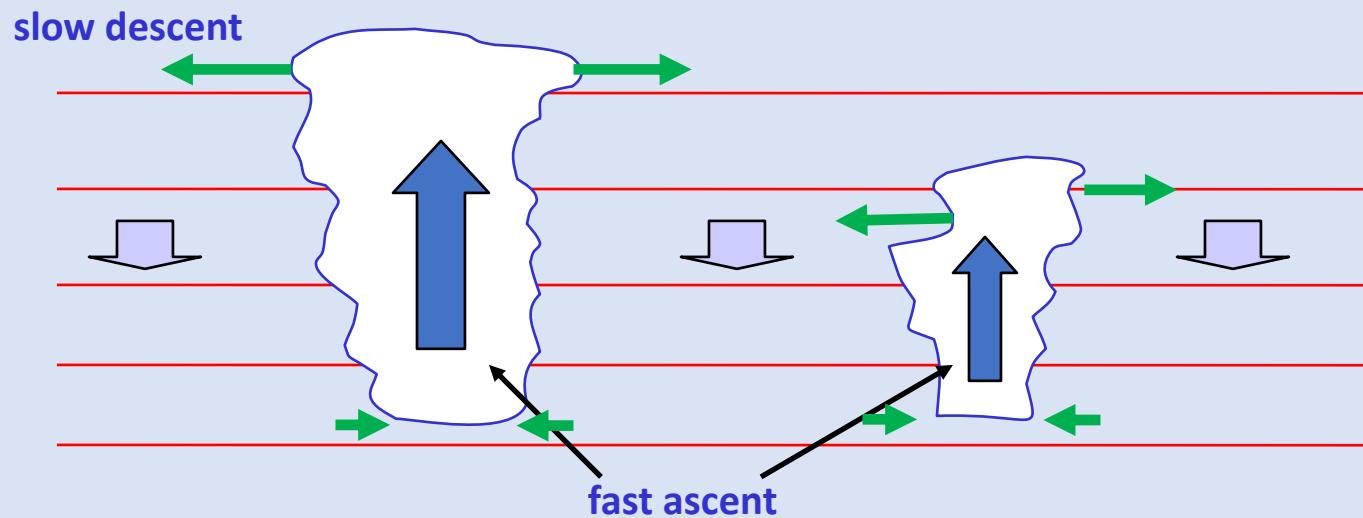


In subsiding regions, adiabatic heating occurs. Because downward motion dries the column, more radiative cooling occurs.

In convection, adiabatic cooling occurs, but is countered by latent heat release. The radiative impacts of clouds are more complicated to quantify.

Statistically on time scales longer than that of individual clouds, convection and the environment are in approximate **radiative-convective equilibrium**.

Subsidence occurs in clear-air outside clouds

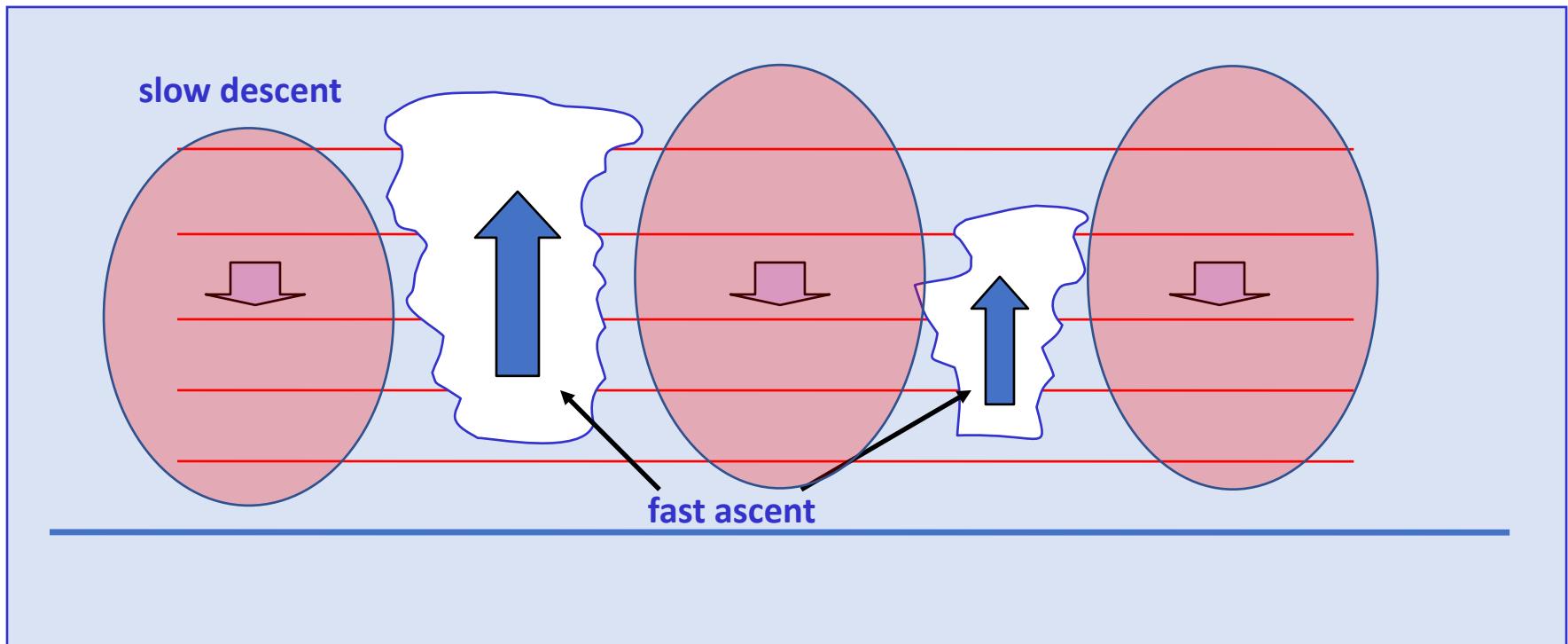


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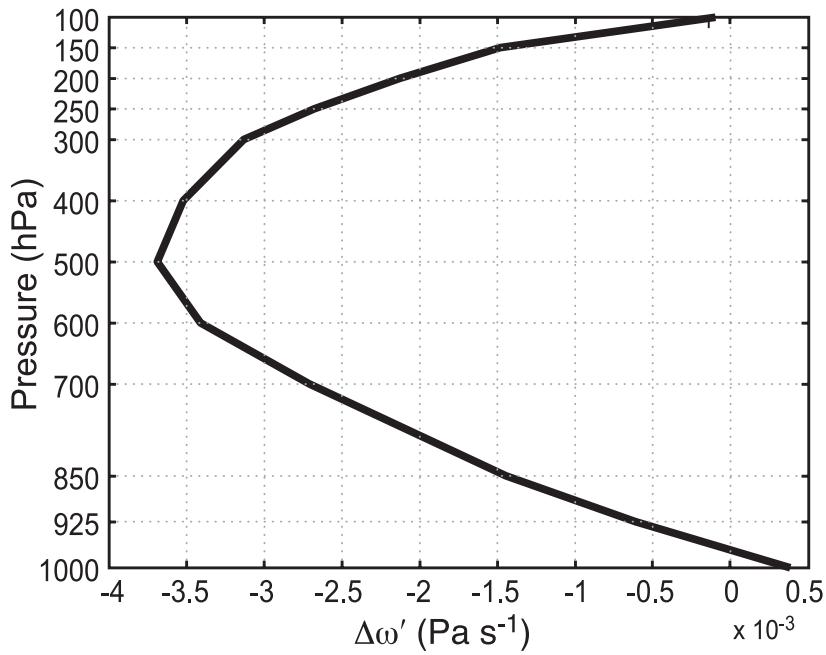
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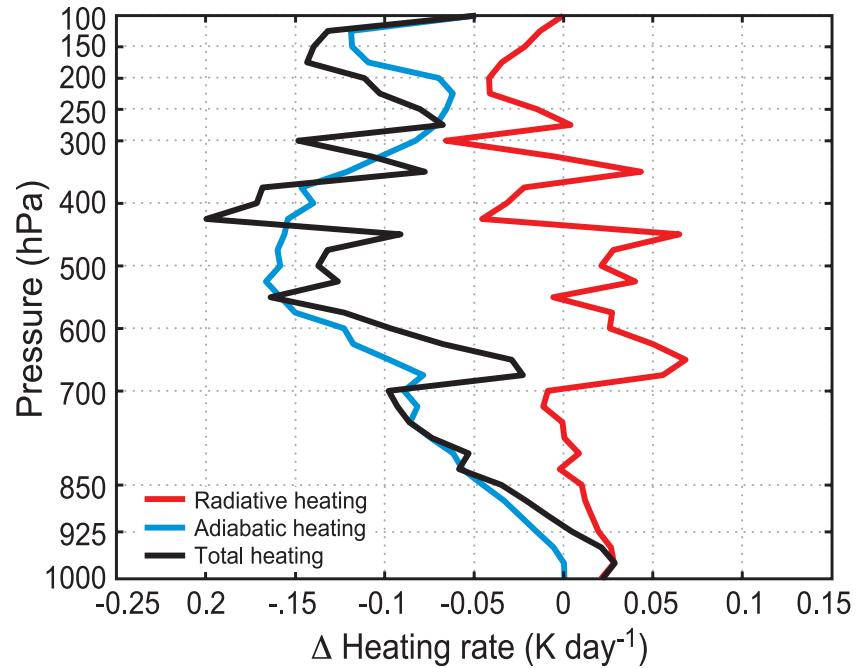
Statistically on time scales longer than that of individual clouds, convection and the environment are in approximate **radiative-convective equilibrium**.

Reducing subsidence can make large-scale environment more conducive to convection.

Change in large-scale vertical motion



Corresponding rate of change in temperature due to adiabatic and radiative processes.



However, do not automatically interpret large-scale upward motion as favorable for convection! It generally means that convection is already present (the upward motion is in the convection itself), and convection reduces conditional instability in the environment.

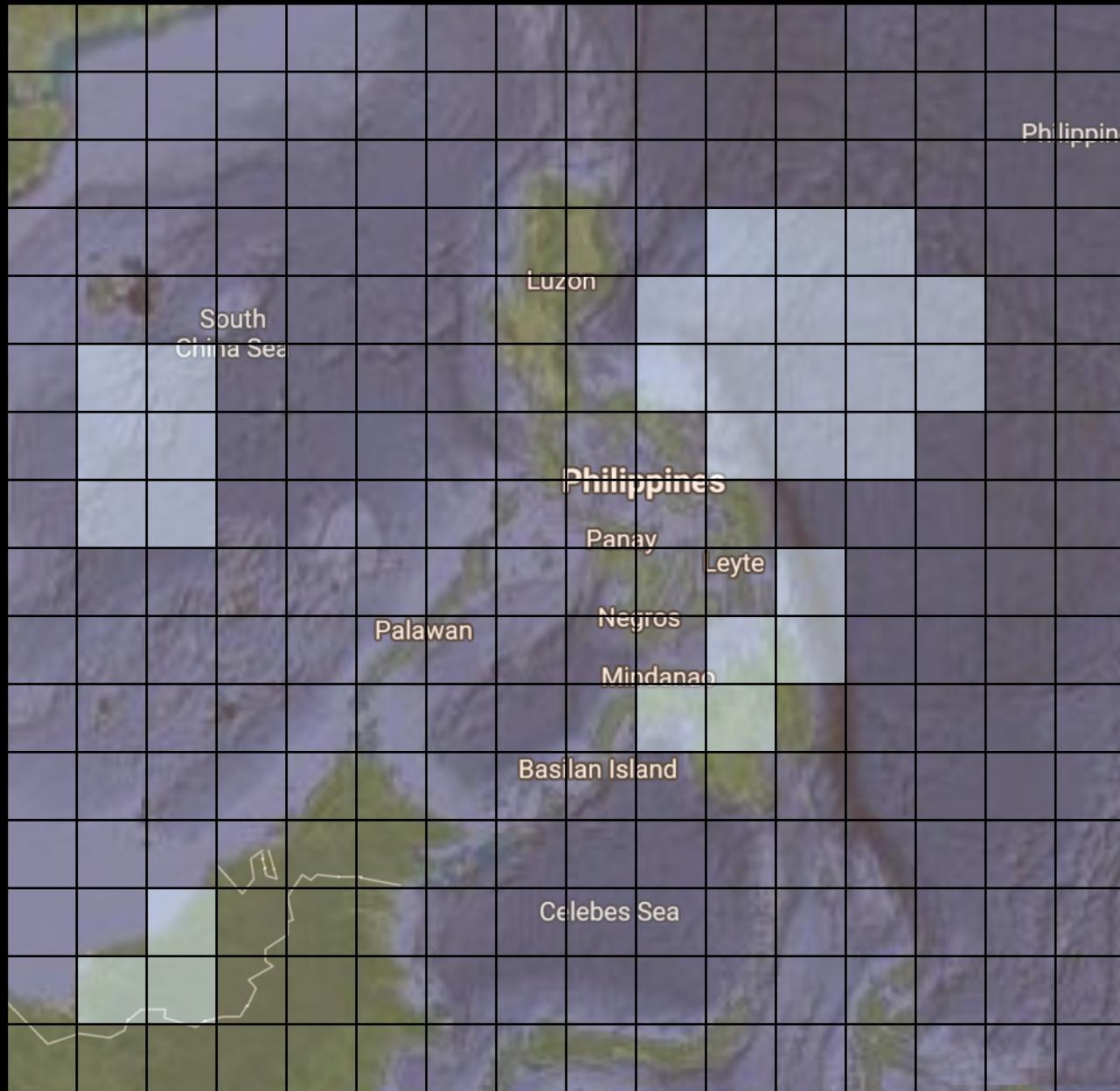
MR3252: Tropical Meteorology

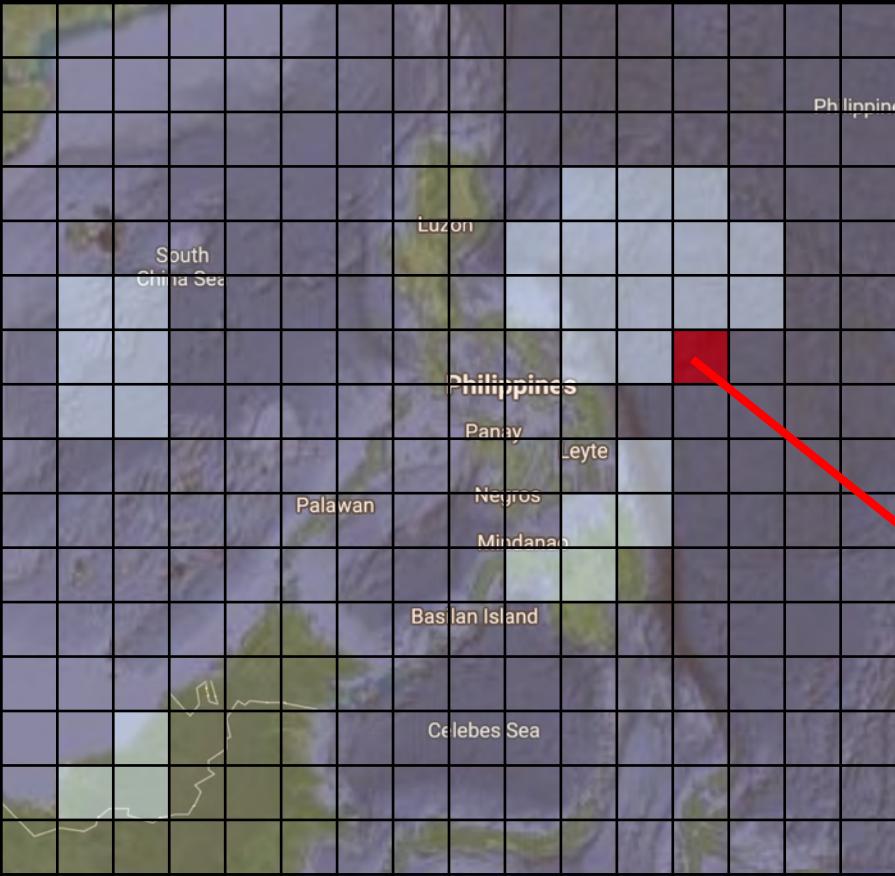
Large-Scale Tropical Heating and Moisture Budgets

Main Topics:

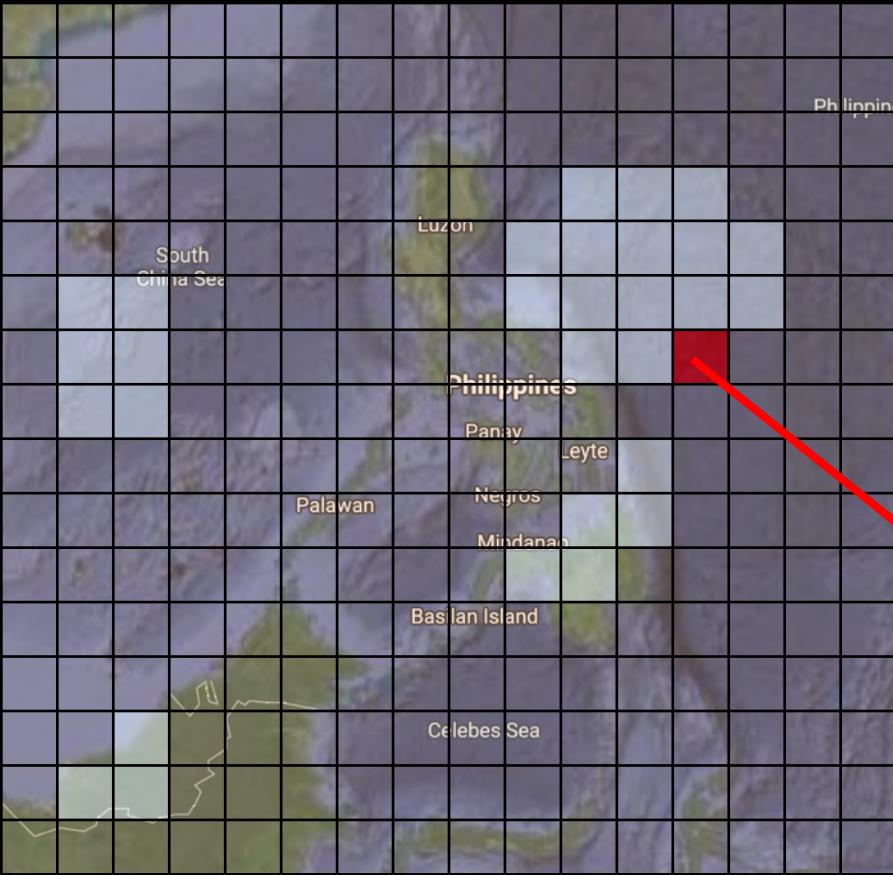
- Large-scale heat sources and moisture sinks
- Relationship of large-scale heating to precipitation and fluxes



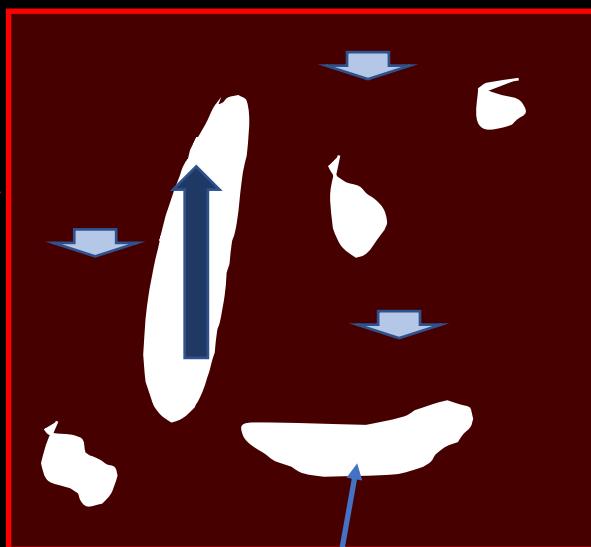




Slices through clouds



$$w = \bar{w} + w'$$
$$w', q', T' \quad \bar{w}, \bar{q}, \bar{T}$$



Slices through clouds

Large-scale budget equations

$$\frac{\partial \bar{s}}{\partial t} + \bar{\nabla} \cdot s\bar{\mathbf{V}} + \frac{\partial \bar{s}\bar{\omega}}{\partial p} = Q_R + Q_c \quad Q_c = L_v(\bar{c} - \bar{e})$$

$$\frac{\partial \bar{q}}{\partial t} + \bar{\nabla} \cdot q\bar{\mathbf{V}} + \frac{\partial \bar{q}\bar{\omega}}{\partial p} = \bar{e} - \bar{c}$$

$$\bar{\nabla} \cdot \bar{\mathbf{V}} + \frac{\partial \bar{\omega}}{\partial p} = 0$$

$s = \text{DSE}$, $h = \text{MSE}$

rate of condensation of water vapor

rate of evaporation of liquid water

Eddy terms describe cloud-scale processes

Apparent heat source (Q_1)

$$Q_1 \equiv \frac{\partial \bar{s}}{\partial t} + \bar{\mathbf{V}} \cdot \Delta \bar{s} + \frac{\partial (\bar{s}\bar{\omega})}{\partial p} = Q_R + Q_c + \frac{\partial}{\partial p} (\bar{\omega}'\bar{s}')$$

Apparent moisture sink (Q_2)

$$Q_2 \equiv -L_v \left(\frac{\partial \bar{q}}{\partial t} + \bar{\mathbf{V}} \cdot \Delta \bar{q} + \frac{\partial (\bar{q}\bar{\omega})}{\partial p} \right) = Q_c + L_v \frac{\partial}{\partial p} (\bar{\omega}'\bar{q}')$$

$* - L_v$

Large-scale budget equations

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$* - L_v$

Large-scale budget equations

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Notice separation of variables beneath mean when defining Q_1 and Q_2 .

$$Q_1 \equiv \frac{\partial \bar{s}}{\partial t} + \bar{V} \cdot \Delta \bar{s} + \frac{\partial (\bar{s} \bar{\omega})}{\partial p} = Q_R + Q_c + \frac{\partial}{\partial p} (\bar{\omega}' s')$$

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Subtract Q_2 from Q_1 :

$$Q_1 - Q_2 = Q_R - \frac{\partial}{\partial p} (\bar{\omega}' h')$$

and

$$Q_1 - Q_2 - Q_R = -\frac{\partial}{\partial p} (\bar{\omega}' h') \longrightarrow$$

Vertical divergence of the vertical eddy transport of MSE
(i.e. mostly in-cloud vertical transport of MSE)

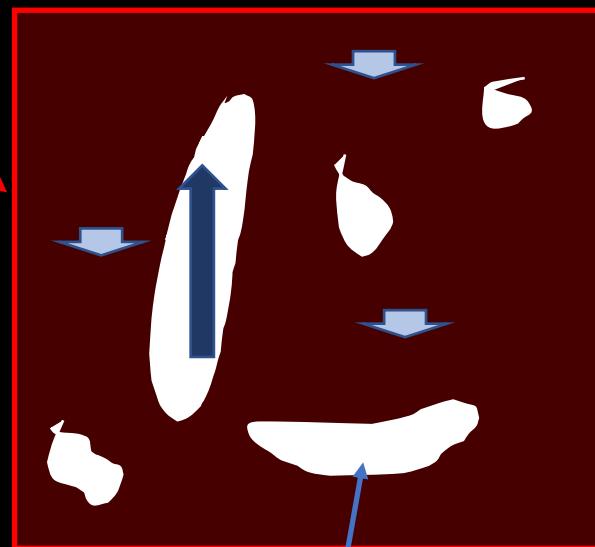
If there is no convection in the area of interest, then $Q_1 - Q_2 = Q_R$



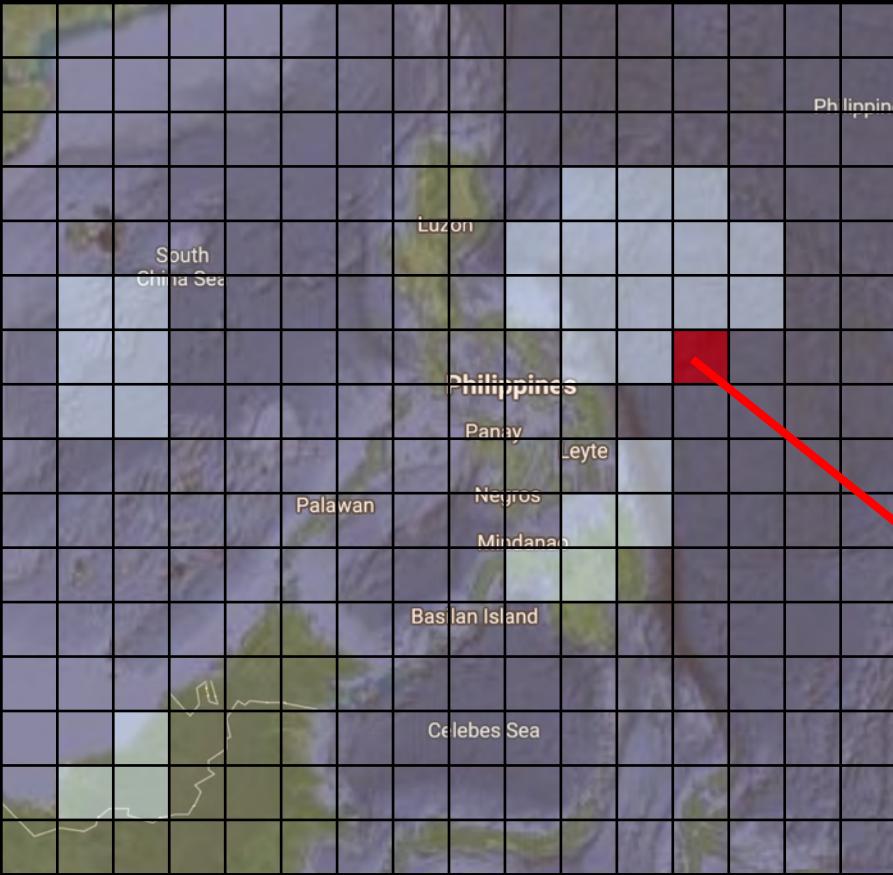
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$$w = \bar{w} + w'$$

$$w', q', T' \quad \bar{w}, \bar{q}, \bar{T}$$



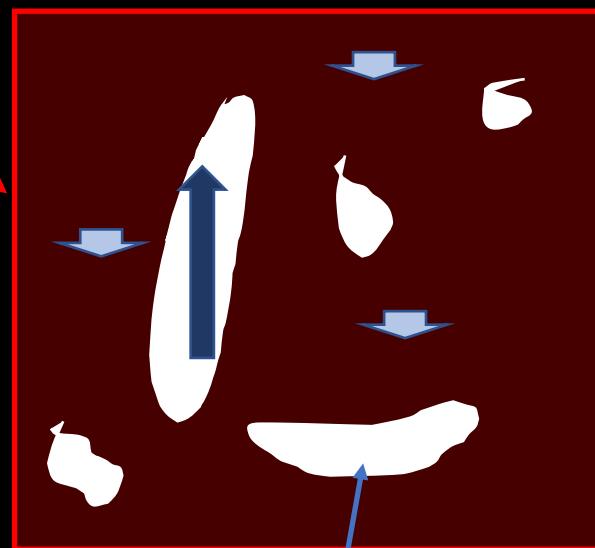
Slices through clouds



$$Q_2 \equiv \frac{\partial \bar{q}}{\partial t} + \bar{V} \cdot \Delta \bar{q} + \frac{\partial(\bar{q}\bar{\omega})}{\partial p} = Q_c + L_v \frac{\partial}{\partial p} (\bar{\omega}'\bar{q}')$$

$$w = \bar{w} + w'$$

$$w', q', T' \quad \bar{w}, \bar{q}, \bar{T}$$

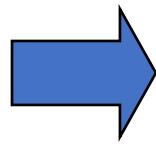


Slices through clouds

Vertically-integrated heat and moisture budgets

$$Q_1 - Q_2 - \bar{Q}_R = -\frac{\partial}{\partial p} (\bar{h' \omega'})$$

Integrate $Q_1 - Q_2 - Q_R = -\frac{\partial}{\partial p} (\bar{h' \omega'})$ **down from the level of cloud tops, p_{top}**

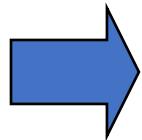


$$\bar{h' \omega'} = \int_{p_{top}}^p (Q_1 - Q_2 - Q_R) dp = -gF$$

F is the vertical flux of total heat.

At the surface, $F_s = SHF + LHF$.

Integrate $Q_1 = Q_R + L_v(\bar{e} - \bar{c}) + \frac{\partial}{\partial p}(\overline{\omega' s'})$ from p_{top} to p_{sfc} (the pressure at the sea surface)



$$\begin{aligned} \frac{1}{g} \int_{p_{top}}^{p_{sfc}} (Q_1 - Q_R) dp &= \frac{L_v}{g} \int_{p_{top}}^{p_{sfc}} (\bar{c} - \bar{e}) dp - \frac{1}{g} (\overline{s' \omega'})|_{p=p_{sfc}} \\ &\approx L_v P + \rho_0 c_p (\overline{T' \omega'})|_{p=p_{sfc}} \\ &= L_v P + SHF \end{aligned}$$

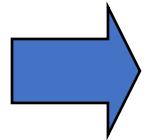
P = Precipitation

SHF = Surface sensible heat flux

Column-integrated radiative heating is the column-integrated heat source minus latent heating due to condensation minus the source of sensible heat from the surface.

Integrate

$$Q_2 = L_v(\bar{e} - \bar{c}) - L_v \frac{\partial}{\partial p} (\overline{\omega' q'}) \quad \text{from } p_{top} \text{ to } p_{sfc}$$



$$\frac{1}{g} \int_{p_{top}}^{p_{sfc}} (Q_2) dp = \frac{L_v}{g} \int_{p_{top}}^{p_{sfc}} (\bar{c} - \bar{e}) dp - \frac{1}{g} (\overline{q' \omega'})|_{p=p_{sfc}}$$

$$\approx L_v P - \rho_0 L_v (\overline{q' \omega'})|_{p=p_{sfc}}$$

$$= L_v(P - E)$$

The rate of evaporation (E)
from the surface, related to
latent heat flux

The four equations:

$$\langle \overline{h' \omega'} \rangle = \int_{p_{top}}^p (Q_1 - Q_2 - Q_R) dp = -gF_s$$

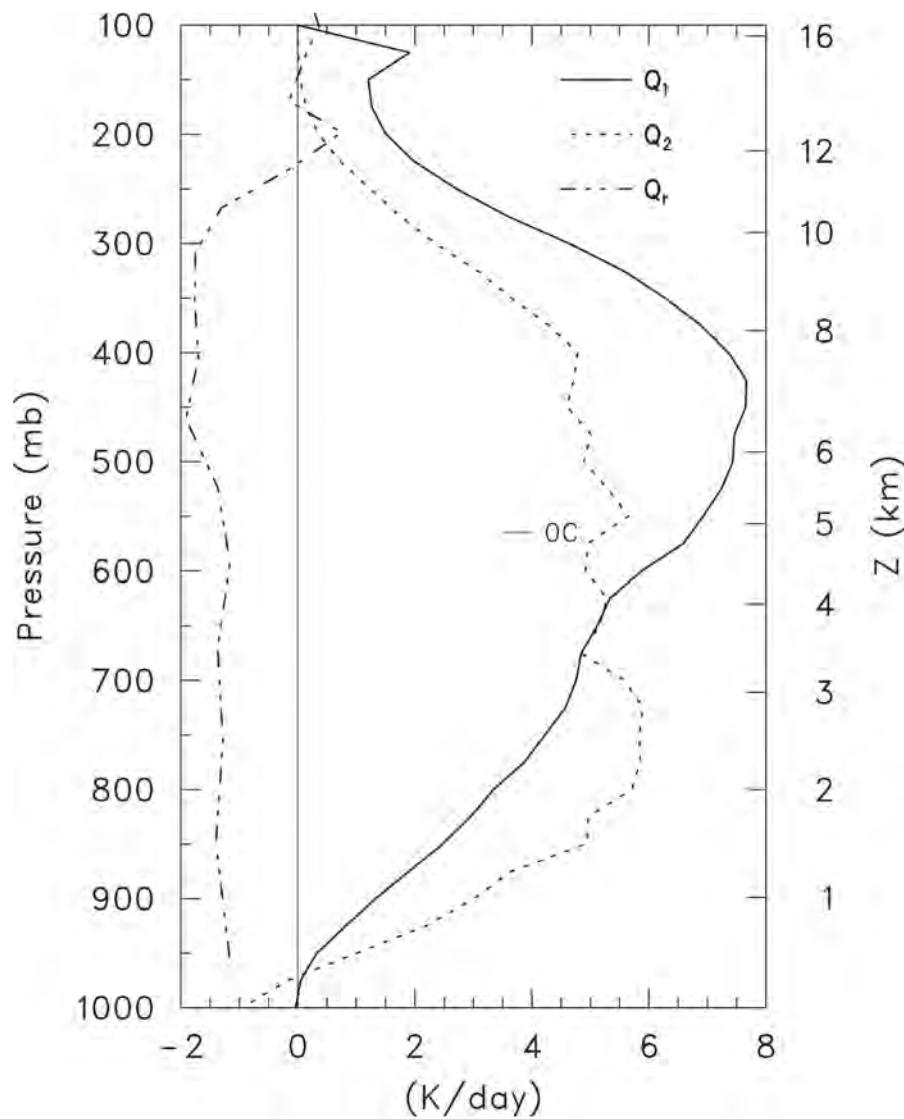
$$\frac{1}{g} \int_{p_{top}}^{p_{sfc}} (Q_1 - Q_R) dp = L_v P + SHF$$

$$\frac{1}{g} \int_{p_{top}}^{p_{sfc}} (Q_2) dp = L_v (P - E)$$

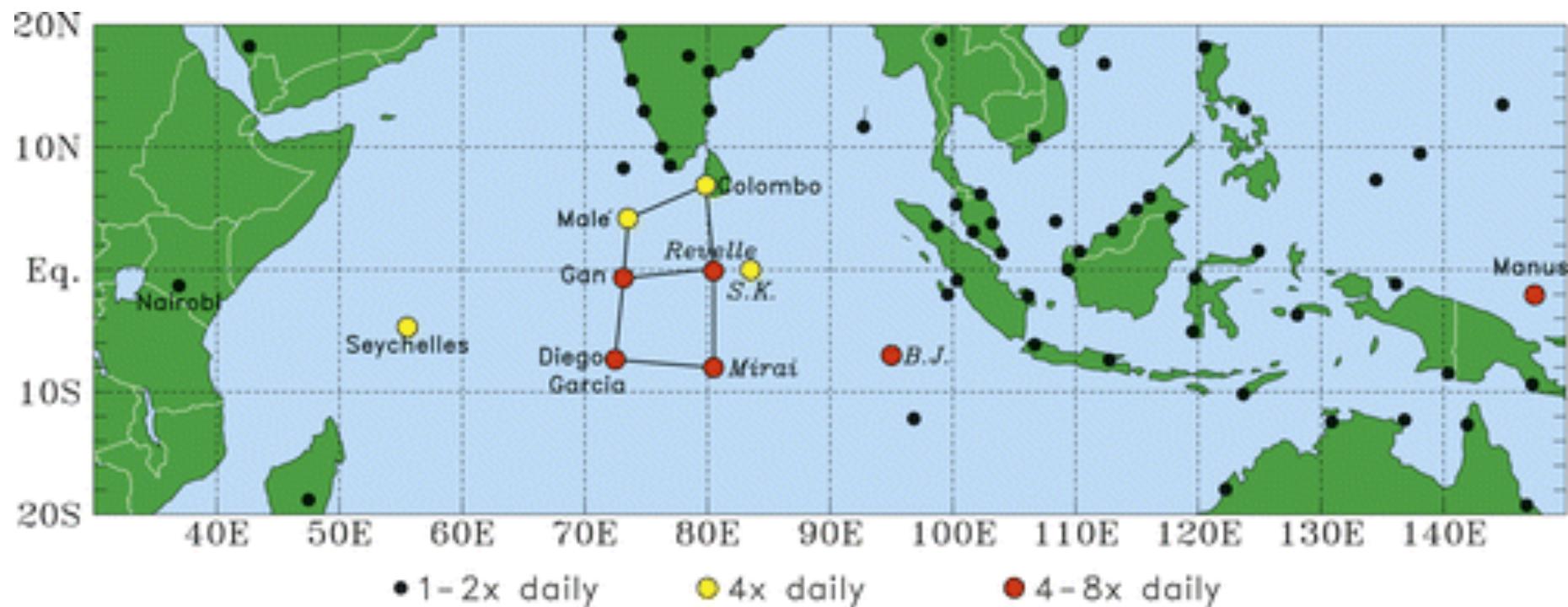
$$F_s = SHF + L_v E$$

show how large-scale heating and precipitation are **constrained by surface fluxes.**

L'Ecuyer and Stephens (2003)



Johnson et al. (2013)



Q₁ and Q₂
together provide
insight for cloud
processes
occurring on
large-scale.

Powell and
Houze (2015)

